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The Variability of Long-duration Rainfall over Great Britain

by R. C. TABONY, B.Sc.

LONDON: HER MAJESTY'S STATIONERY OFFICE

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by R. C. Tabony, B.Sc.

SUMMARY

This paper presents a technique for calculating the return periods of rainfall excesses or deficiencies, measured over n months starting in month m, for any point or area in Great Britain. Reliable values of the coefficient of variation (C_V) and the coefficient of skewness (C_S) are obtained from point rainfall records for a duration of one month. Values of C_V and C_S for longer durations are derived from the values for one month by means of relations based on the independence of consecutive rainfall totals. Extensive data are used to develop equations which permit the values of C_V and C_S appropriate to points to be extended to areas. A log-normal distribution using C_V and C_S as input parameters is then used as a frequency distribution from which rainfall deficiencies and excesses corresponding to given return periods are calculated.

1. INTRODUCTION

An adequate knowledge of rainfall climatology is essential for agriculture, water management and many other important activities; a good description thereof, for a place, is provided by:

(a) the average annual amount,

(b) the average seasonal distribution, and

(c) the month-to-month and year-to-year variations.

The average annual rainfall of Great Britain and its seasonal distribution have been well documented. The variability of point rainfall over the country has been examined by Glasspoole (1922), Gregory (1955, 1957), Bleasdale *et alii* (1963) and Senior (1969) but still more work is needed to support water resource planning. The required statistics are the return periods* of rainfall deficiencies (or excesses) lasting, say, a month or more, not merely at point locations, but over whole catchments. Investigation of the variability of areal rainfall presents more complex problems than arise in analyses of point falls. In this study the conventional and logical approach of proceeding from point to areal frequencies is followed. Areal rainfall events of given return periods are derived from point events of the same probabilities by means of 'areal reduction factors'. Most past work has been concerned with flood-producing rainfall with a time-scale of minutes, hours or days. Areal reduction factors for those falls are derived in Volume II of the *United Kingdom*

*A rainfall deficiency (excess) with rainfall R has a return period of T years if there is a probability of $1/T$ that a rainfall deficiency (excess) with rainfall less (greater) than or equal to R will start in any one year.

Flood Studies Report (Meteorological Office, 1975). Depth–area relationships for the United States are given by Hershfield (1961, 1967). The US Weather Bureau areal reduction formula is reported by Leclerc and Shanke (1972) and a good general discussion is provided by Rodriguez-Iturbe and Mejia (1974). Smith (1974) discusses areal reduction as applied to thunderstorms. Roche (1963) provides a general theoretical treatment of the problem but his methods are not readily applicable in practice.

In the first part of the project now described (see Section 5) a method is developed for calculating relationships between deviations of accumulated point rainfall from average and return periods. Section 6 reports the derivation of an areal reduction technique. Volume II of the *UK Flood Studies Report* provides an objective method for answering enquiries relating to flood-producing rainfall, based on an extreme-value distribution and is limited to events lasting up to 25 days. With the main aim of answering enquiries on drought, the present work has involved the complete rainfall distribution and a far longer time-scale.

2. DATA

The basic body of data consisted of monthly rainfall totals for the period 1911–70 from 131 stations in England and Wales. Of these stations, 96 are uniformly distributed; the other 35 form four high-density networks (London, Bath area, industrial Lancashire and North Yorkshire). Records were complete for 102 stations; of the remaining 29, a few were deficient of up to 10 per cent of data and about half had gaps of one year or less. These gaps were filled by making estimates from nearby stations, adjusting for differences in average annual rainfall ($\bar{R}_A$). The basic data were supplemented by monthly totals for:

(a) 32 wet stations ($\bar{R}_A > 1500$ mm) with 40–60-year records,

(b) some 2000 stations in Great Britain for the period 1941–70, and

(c) three long-period sets of monthly totals representative of (1) England and Wales from 1727, constructed by Nicholas and Glasspoole (1932) and since kept up to date by the Meteorological Office, (2) Manchester from 1786, prepared by Manley (1973) and (3) Kew from 1697 assembled by Wales-Smith (1971).

3. COMPARISON OF 1911–70 WITH EARLIER PERIODS

Figure 1 shows monthly average rainfalls for 60-year periods and from 1731 (1791 for Manchester) from the three series already mentioned. Dry springs and wet autumns are persistent features. It should be noted that the England and Wales series has been compiled from an increasing and more generally representative set of stations over the years. Northern and western districts in particular were poorly represented in the 18th century. This is reflected in the rising annual rainfall and decreasing proportion thereof occurring in the summer half-year in England and Wales as compared with Manchester and Kew.

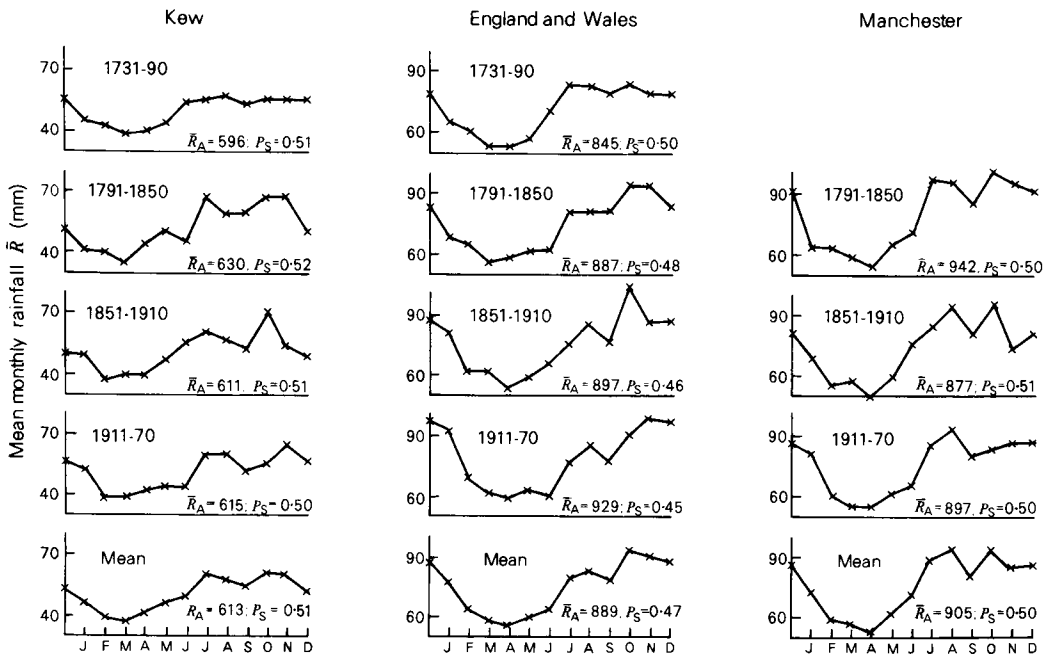

FIGURE 1. Seasonal distribution of rainfall

$\bar{R}_A$ denotes average annual rainfall (AAR) in millimetres; P_s denotes proportion of summer half-year rainfall.

A detailed examination of the distribution of rainfall about its mean was made. From N years of record monthly rainfall totals summed over a number of months and starting with a given month were arranged in ascending order. The probability M_P that the rainfall M_R, associated with the mth rank, will be exceeded was calculated from

$$M_P = \frac{1}{M_T} = \frac{M - 0.31}{N + 0.38} \quad \text{(Jenkinson, 1969)}$$

where M_T is the return period associated with M_P. Linear interpolation between adjacent values of M_R gave rainfall totals corresponding to given return periods. Figures 2 and 3 show percentages of average rainfall for M_T = 25 years, for rainfall deficiencies starting with a given month and lasting for one month (only) and for 12 months, respectively. The percentages vary widely with starting month and between periods of record. It is clear that 60 years of data and the simple procedure described above do not yield stable results even for return periods as short as 25 years. To give an overall view of the rainfall distribution the coefficients of variation (C_V) and of skewness (C_S) – defined in the Appendix – were calculated for each month for 60-year periods and from 1731 or 1791 for the three long series. Figure 4 shows a mean maximum of C_V in spring and a mean minimum in autumn. The seasonal variations in individual 60-year periods are different from one another and from the means and show the existence of large random components. Figure 5 shows that values of C_S are very unstable with no persistence from season to season or in monthly means.

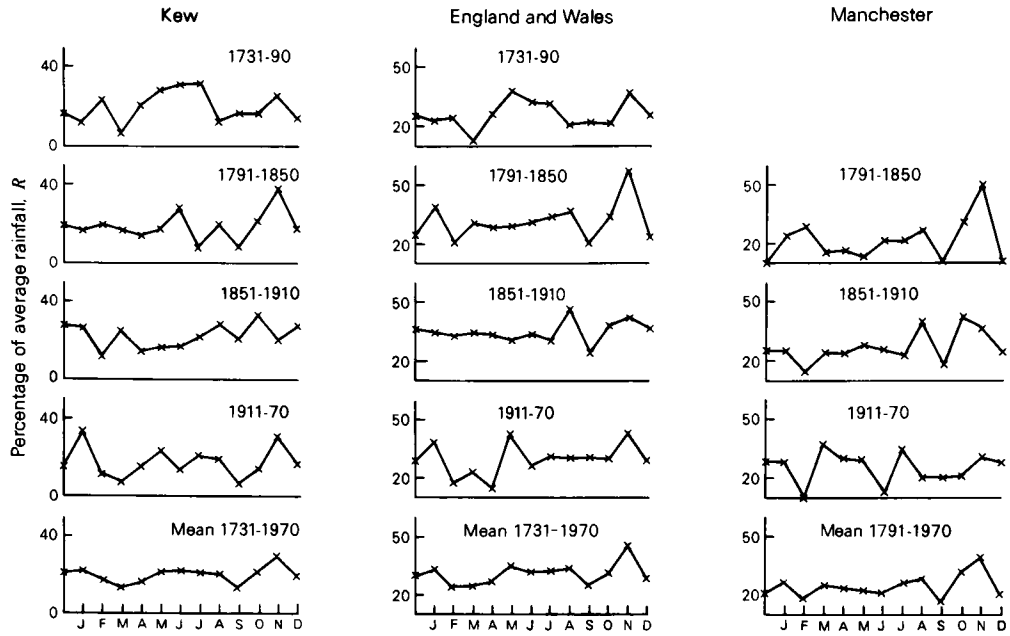

FIGURE 2. Percentage of average rainfall corresponding to a '25-year' rainfall deficiency for a duration of one month

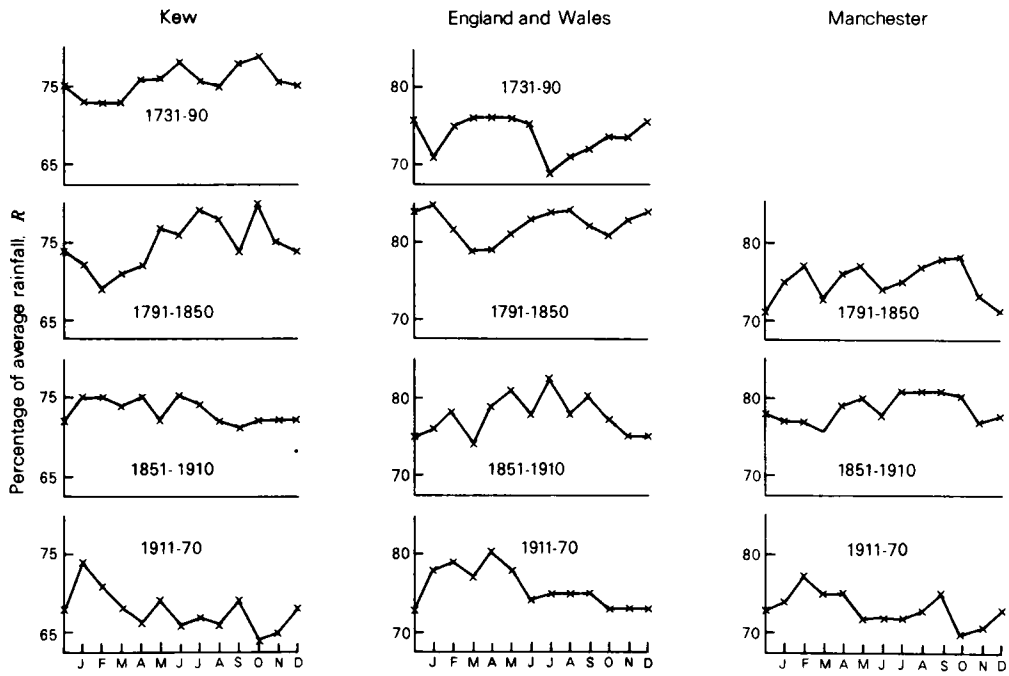

FIGURE 3. Percentage of average rainfall corresponding to a '25-year' rainfall deficiency for a duration of 12 months

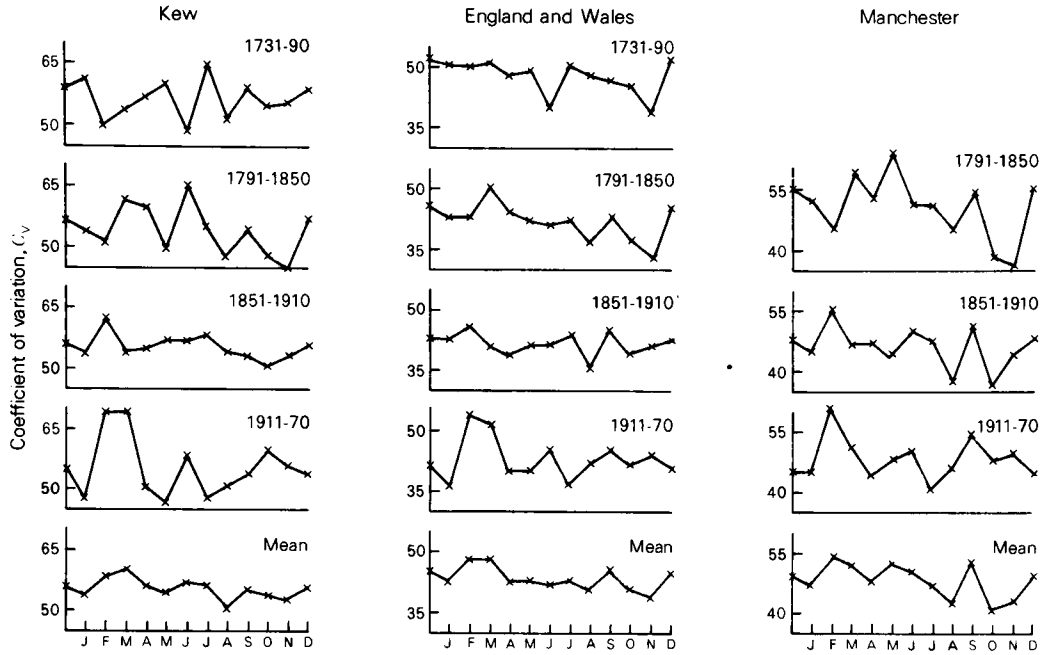

FIGURE 4. Seasonal distribution of coefficient of variation of monthly rainfall

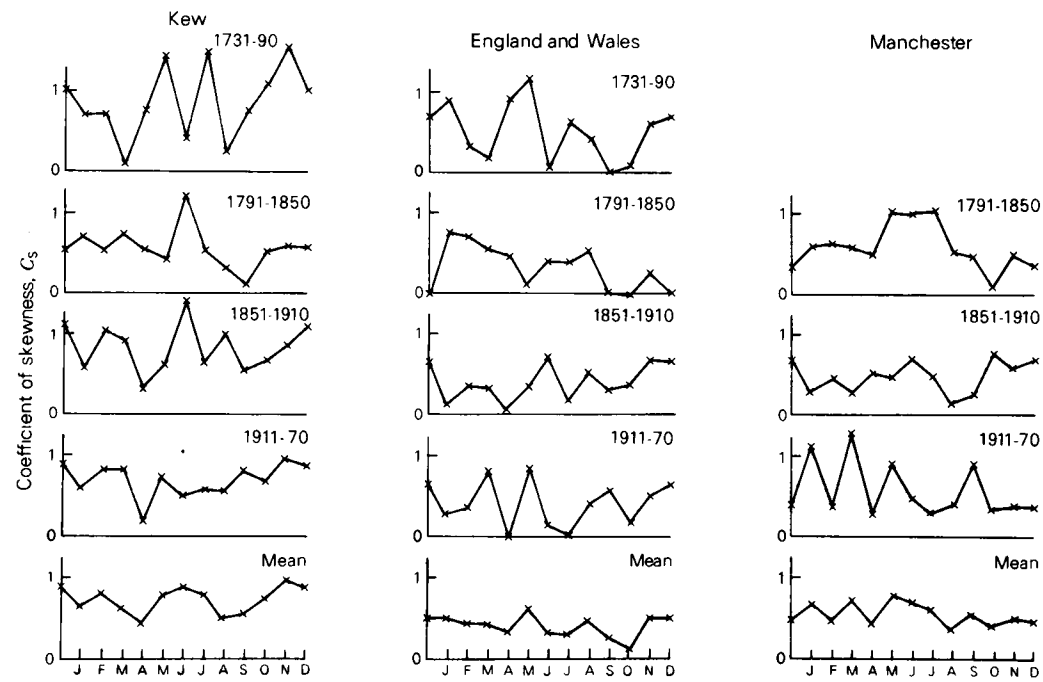

FIGURE 5. Seasonal distribution of coefficient of skewness of monthly rainfall

It is clear that C_v and C_s for individual months form no marked seasonal patterns and are not stable from one period of record to another. To obtain more stable values C_v and C_s were meaned over all starting months for various numbers of months; some results are shown in Tables I and II. Confidence in values decreases with increasing duration as the number of cases per 60 years decreases. For a nearly normal distribution, Brooks and Carruthers (1953) state that the standard errors E_v and E_s (of C_v and C_s) are given by:

$$E_v = (0.5C_v + 10^{-4}\ C_v{}^3)/\sqrt{N} \text{ and } E_s = \sqrt{(6/N)}$$

where N is the number of independent cases.

For 60 years $E_v \approx 0.01$ for all durations whereas E_s ranges from 0.09 for one month to 0.33 for 12 months and 0.55 for 36 months. Random variations would result in figures within about 2 standard errors of the mean. For Kew values of C_v and C_s were obtained over 60-year periods for 6 durations starting in each decade (in addition to those shown in Tables I and II). All values lay within about 2 standard errors of the mean, implying that observed ranges of values could have occurred by chance.

TABLE I. *Coefficient of variation of rainfall (meaned over all starting months)*

Kew	Duration (months)					
Period	1	3	6	12	24	36
1731–1790	0.564	0.332	0.225	0.154	0.108	0.086
1791–1850	0.541	0.303	0.213	0.154	0.112	0.099
1851–1910	0.555	0.333	0.259	0.188	0.132	0.099
1911–1970	0.553	0.327	0.239	0.176	0.128	0.106
England and Wales	Duration (months)					
Period	1	3	6	12	24	36
1731–1790	0.474	0.281	0.193	0.135	0.098	0.081
1791–1850	0.417	0.235	0.158	0.107	0.070	0.056
1851–1910	0.416	0.243	0.185	0.144	0.102	0.080
1911–1970	0.432	0.256	0.183	0.131	0.088	0.066
Manchester	Duration (months)					
Period	1	3	6	12	24	36
1791–1850	0.504	0.292	0.201	0.142	0.104	0.086
1851–1910	0.462	0.266	0.194	0.143	0.104	0.086
1911–1970	0.485	0.274	0.198	0.145	0.100	0.078

TABLE II. *Coefficient of skewness of rainfall (meaned over all starting months)*

Kew	Duration (months)					
Period	1	3	6	12	24	36
1731–1790	0.85	0.45	0.52	0.38	−0.01	−0.16
1791–1850	0.56	0.34	0.82	0.24	0.47	0.60
1851–1910	0.81	0.66	0.78	0.81	0.69	0.72
1911–1970	0.67	0.30	0.01	−0.18	−0.11	−0.03

England and Wales	Duration (months)					
Period	1	3	6	12	24	36
1731–1790	0.49	0.15	0.15	0.15	−0.08	−0.33
1791–1850	0.34	0.27	0.40	0.30	0.17	0.21
1851–1910	0.40	0.24	0.39	0.52	0.58	0.80
1911–1970	0.39	0.25	0.05	−0.09	−0.21	−0.41

Manchester	Duration (months)					
Period	1	3	6	12	24	36
1791–1850	0.61	0.36	0.25	0.28	0.10	−0.16
1851–1910	0.58	0.39	0.41	0.48	0.60	0.55
1911–1970	0.60	0.24	0.17	−0.08	0.05	0.07

4. GENERAL RELATIONSHIPS OF C_v AND C_s TO DURATION, MEAN RAINFALL AND AREA

The standard errors of the estimates of C_v and C_s obtained for a duration of one month and ignoring any seasonal effects, are small enough to yield stable values of C_v and rather less stable values of C_s from 60 years of monthly rainfall totals. On this basis a distribution could be sought which would model rainfall return period relationships based on the input of C_v and C_s of monthly rainfall (for all months). From these 1-month values (of C_v and C_s) obtained for point locations values for n months and for areas of specified sizes would have to be calculated. The expected responses of C_v and C_s to changes of period, mean rainfall and area are described briefly below.

4.1. *Variation of C_v and C_s with duration*

If at a location there are no seasonal differences in rainfall characteristics and if monthly rainfall totals are independent of one another then C_v over n months $C_{v,n}$ is related to $C_{v,1}$ by the equation

$$C_{v,n} = C_{v,1}/\sqrt{n}. \qquad \ldots (1)$$

A similar relationship holds for C_s (Brooks and Carruthers, 1953, page 55).

Consecutive daily rainfall totals show some degree of persistence, but this quality fades with increasing period of duration. Detailed results are presented later and from these it can be stated that, to a very good approximation, equation (1) expresses the behaviour of C_v in Great Britain for periods of duration of one month or more.

4.2. *Variation of C_v and C_s with mean rainfall*

As well as being dependent on duration C_v may also be related to mean rainfall and this relationship may change with duration. Any change in the relationship with duration will involve differences in the rate of decrease of C_v and C_s with duration between wet and dry locations. It is therefore necessary to consider how C_v and C_s might vary with $\bar{R}$, the mean rainfall. For long durations C_v and C_s may be expected to be independent of $\bar{R}$ but an inverse relationship is to be expected for short durations. This may be expressed by writing $C_v \propto 1/\bar{R}^{\beta}$, where $0 < \beta < 1$ ... (2)
with a similar relation for C_s; $\bar{R}$ may change from a dry to a wet location, or, at a fixed location, from a dry period to a wet period. β for a change of location will be denoted by β_s and for change in time by β_t.

4.3. *Variation of C_v and C_s with area*

The ratio of C_v over an area, $C_{v,A}$ to that at a point $C_{v,P}$ is unity for $A = 0$ and should decrease with increasing area. A relationship of the form

$$C_{v,A} = C_{v,P} \exp(-\gamma_v A) \qquad \ldots (3)$$

may be expected, γ_v being defined as the areal reduction coefficient of C_v. A similar relationship holds for C_s.

4.4. *Composite behaviour of C_v with duration, mean rainfall and area*

Equations (1), (2) and (3) may be combined to give

$$C_{v\,(n,\bar{R},A)} = C_{v\,(1,r,0)}\,(1/n^{\alpha})\,(nr/\bar{R})^{\beta t} \exp(-\gamma_v A) \ldots (4)$$

where n is duration in months,

r is mean monthly rainfall averaged over all months (i.e. $\bar{R}_A/12$),

nr is mean rainfall over n months averaged over all starting months (i.e. $\bar{R}_A \times n/12$)

$\bar{R}$ is mean rainfall during the n-month period,

A is the area,

α measures the decrease of C_v with increasing n,

β_t measures the decrease of C_v with increasing $\bar{R}$, and

γ_v measures the decrease of C_v with increasing A.

Equation 4 will only be valid if the variations of C_v with n, $\bar{R}$ and A are independent of one another. As this may not be true the general case is covered by assuming that

$$\alpha = \alpha(\bar{R}) \quad \ldots(5) \qquad \alpha = \alpha(A) \quad \ldots(9)$$
$$\beta_t = \beta_t(n) \quad \ldots(6) \qquad \beta_t = \beta_t(A) \quad \ldots(10)$$
$$\beta_s = \beta_s(n) \quad \ldots(7) \qquad \beta_s = \beta_s(A) \quad \ldots(11)$$
$$\gamma_v = \gamma_v(n) \quad \ldots(8) \qquad \gamma_v = \gamma_v(\bar{R}) \quad \ldots(12)$$

β_s has been included although it does not appear explicitly in equation (4). The various relationships implied by equation (4) and expressed in relationships (5) to (12) are illustrated schematically in Figures 6a to 6c. The following implications follow from assuming the stated relationships:

Figure 6a	$\beta_t = \beta_t(n)$ or $\beta_s = \beta_s(n)$	implies	$\alpha = \alpha(\bar{R})$	$\ldots(13)$
Figure 6b	$\beta_t = \beta_t(A)$ or $\beta_s = \beta_s(A)$	implies	$\gamma_v = \gamma_v(\bar{R})$	$\ldots(14)$
Figure 6c	$\gamma_v = \gamma_v(n)$	implies	$\alpha = \alpha(A)$.	$\ldots(15)$

So far this section has been mainly limited to a discussion of C_v, but similar relationships are expected to apply to C_s from simple reasoning. A rational and reliable assessment of some of the rainfall characteristics of a location must rely upon very long, good quality records (if such are obtainable) and should be in accord with physical principles. The latter specify relationships between C_v and C_s, geographical position and topography, and these relationships do appear when values of the parameters (for all monthly rainfalls) are mapped for a large number of places and isopleths are drawn. The signal-to-noise ratio of rainfall data is low and the resulting errors associated with C_s are much larger than those associated with C_v. For this reason, C_v only is analysed in this investigation and from first principles C_s is assumed to behave like C_v.

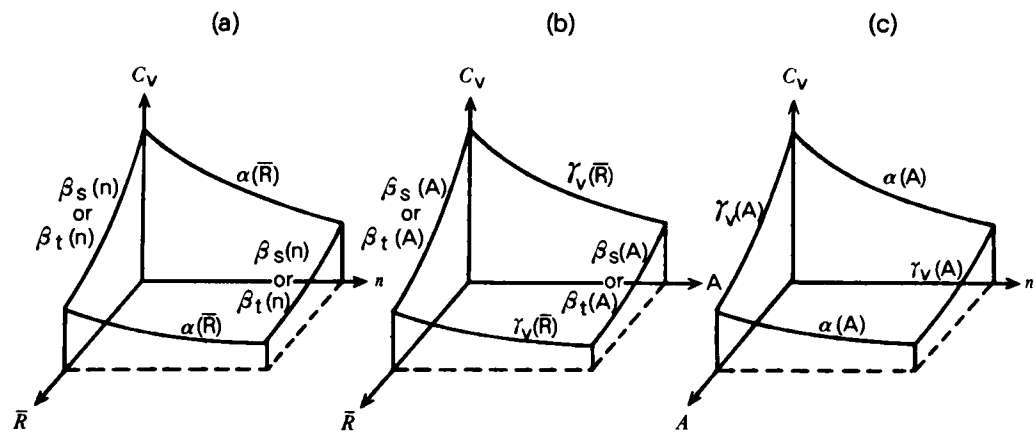

FIGURE 6. Relationships between coefficient of variation of rainfall and duration, mean rainfall, and area

5. THE VARIABILITY OF POINT RAINFALL

For point rainfall, equation (4) reduces to

$$C_{v(n,\bar{R})} = \frac{C_{v(1,r)}}{n^{\alpha}} \cdot \left(\frac{nr}{\bar{R}}\right)^{\beta t}$$

with $\alpha = \alpha(R)$, $\beta_s = \beta_s(n)$ and $\beta_t = \beta_t(n)$ as the relationships to be investigated.

5.1. *Estimation of* α

In 4.1 it was seen that if consecutive monthly rainfall totals are independent of one another and if there are no seasonal variations of C_v, the value of α may be expected to be 0.5. Figures 4 and 5 show that seasonal variations in C_v and C_s (of monthly rainfall) are very small.

For rainfall summed over a number of months Murray (1967) and Stephenson (1967) failed to find any significant persistence over England and Wales. In the present study a mean correlation coefficient of 0.014 was obtained between one month's rainfall and the next at 96 stations, uniformly distributed over England and Wales, for the period 1911–70. All values were close to zero and it is concluded that monthly rainfalls may be regarded as being (sequentially) independent of one another. Hence the assumption $\alpha = 0.5$ would be reasonable on the two counts specified in 4.1. If there is an element of climatic change, however, resulting in an increase in the variability, the effect increasing with duration, α will be depressed below 0.5. To investigate this, α was calculated for the uniformly distributed set of 96 stations for 1911–70. Starting in each month for the year C_v and C_s were calculated for rainfall summed over n months, for n ranging from 1 to 120. Values obtained were meaned over all starting months and the variation of $C_{v,n}$ with n was examined. A relationship of the form

$$C_{v,n} = \frac{C_{v,1}}{n^{\alpha}} \qquad \ldots (16)$$

was assumed, where α is the slope of a straight line on a graph of log $(C_{v,n})$ against log n. Results from 3 stations are presented in Figure 7. The straight lines correspond to $\alpha = 0.5$. Considering the size of the errors involved it can be seen that equation (16) with $\alpha \approx 0.5$ gives a reasonable fit to the data. α was then calculated for each of the 96 stations, using a least squares method which weighted each value according to the magnitude of the standard errors involved. The 1911–70 96-station mean, for n ranging from 1 to 120, was $\alpha = 0.493$, with a standard deviation of 0.03. The value of α determined from the long series for Kew was 0.45. If n had been limited at 12 the mean α would have been 0.480. Accordingly, to secure the best agreement with available observations α has been taken as 0.48.

In order to investigate the possible variation of α with average annual rainfall, $\bar{R}_A$, the calculated values of α for the 96 stations, for the period 1911–70, supplemented by values calculated for 32 wet stations (in Scotland, the Lake District, Wales and south-west England) for shorter periods of record, were plotted against average annual rainfall. The correlation between α and $\bar{R}_A$ was 0.002. Hence it was concluded that α is independent of $\bar{R}_A$ and that observed variations can be attributed to chance.

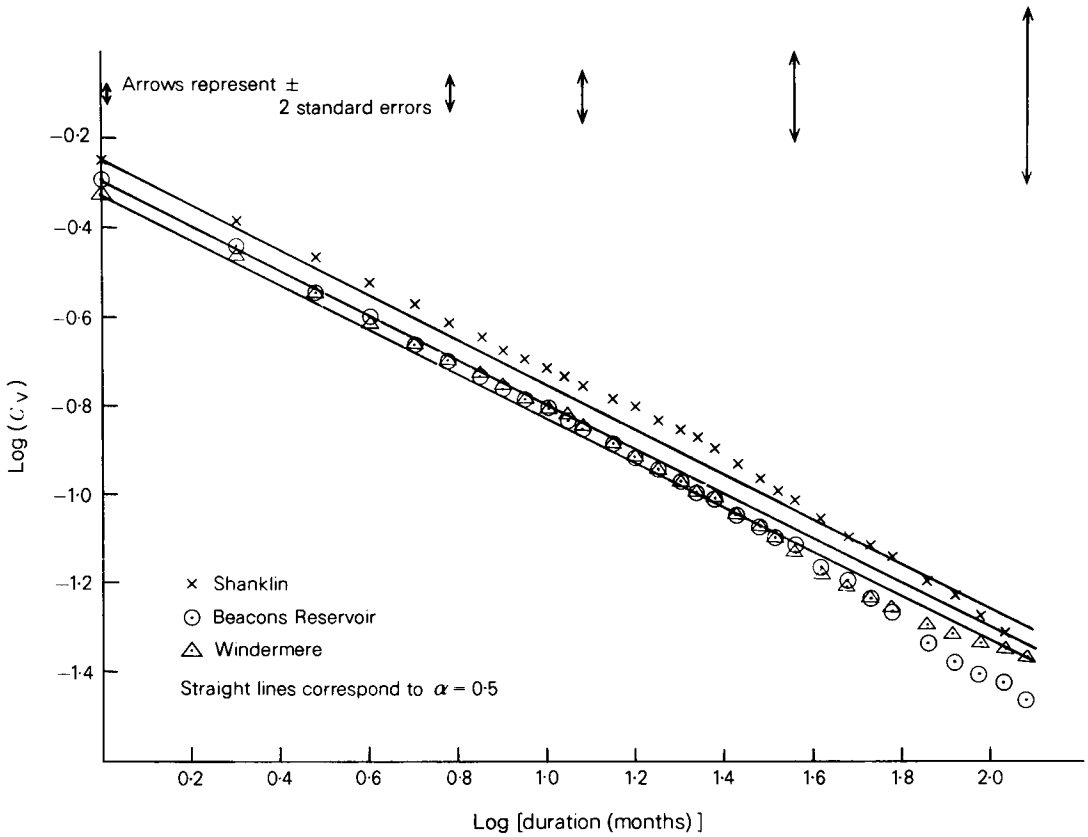

FIGURE 7. Coefficient of variation of rainfall against duration for three typical stations

5.2. *Estimation of* β_s

β_s is essentially a measure of the variation of C_v (and C_s), meaned over all months, with $\bar{R}_A$. Complications are introduced, however, because any observed changes of C_v with $\bar{R}_A$ may be due to geographical location.

Maps of C_v and C_s of monthly rainfall over Great Britain were constructed for the period 1941–70 from about 2000 station calculations. The isopleth patterns, presented in Figures 8 and 9, suggest the following interpretation:

(a) Low values occur to the prevailing windward and high values occur to the prevailing leeward of high ground;

(b) Values increase with 'continentality'; and

(c) A superimposed random component.

C_v of monthly rainfall, meaned over all months, for the 96 stations already mentioned, plotted against $\bar{R}_A$ showed an inverse relationship. For annual rainfall Rao *et alii* (1972) in India and Hershfield (1962) in the United States also found that C_v decreased with

increasing $\bar{R}_A$. The data from the 32 wet stations, however, did not show any decrease of C_v with $\bar{R}_A$. The observed variation of C_v with $\bar{R}_A$ may be interpreted geographically. Higher C_v will be associated with the relatively 'continental' south-east and lower C_v with the maritime north-west. The south-east is drier than the north-west and thus the 96 uniformly distributed stations show decreasing C_v with increasing $\bar{R}_A$. The wetter stations used are all in the west and north and thus geographical differences in C_v are not to be expected and are not found. It is concluded from the foregoing results that C_v is independent of $\bar{R}$ and that $\beta_s = 0$.

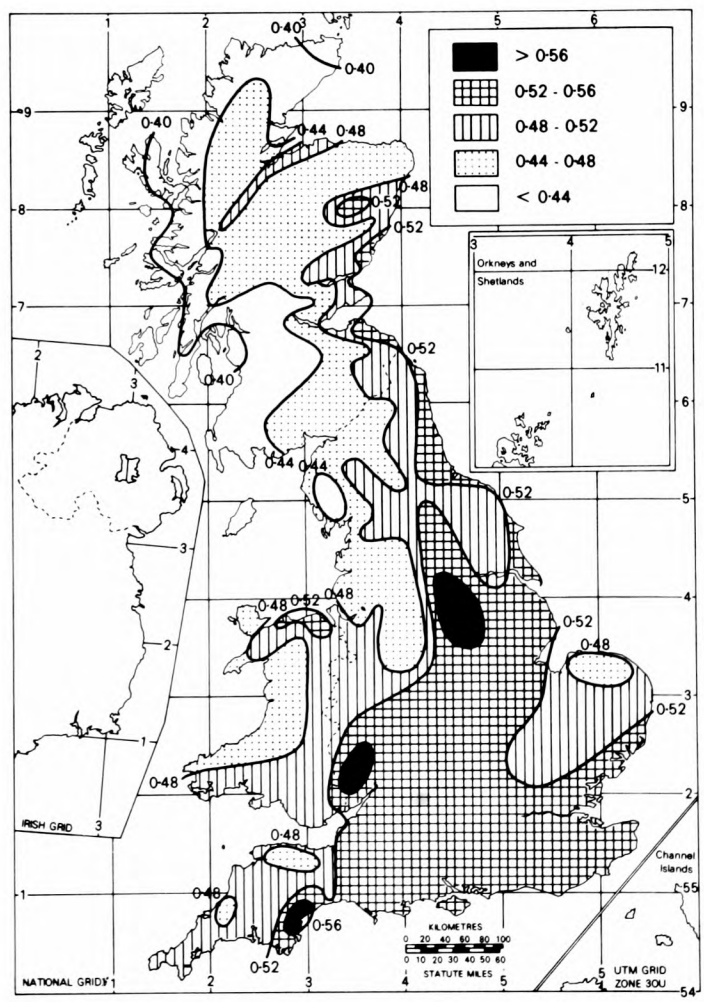

FIGURE 8. Coefficient of variation of monthly rainfall from 1941 to 1970 (mean of all months)

Standard error ≈ 0.013.

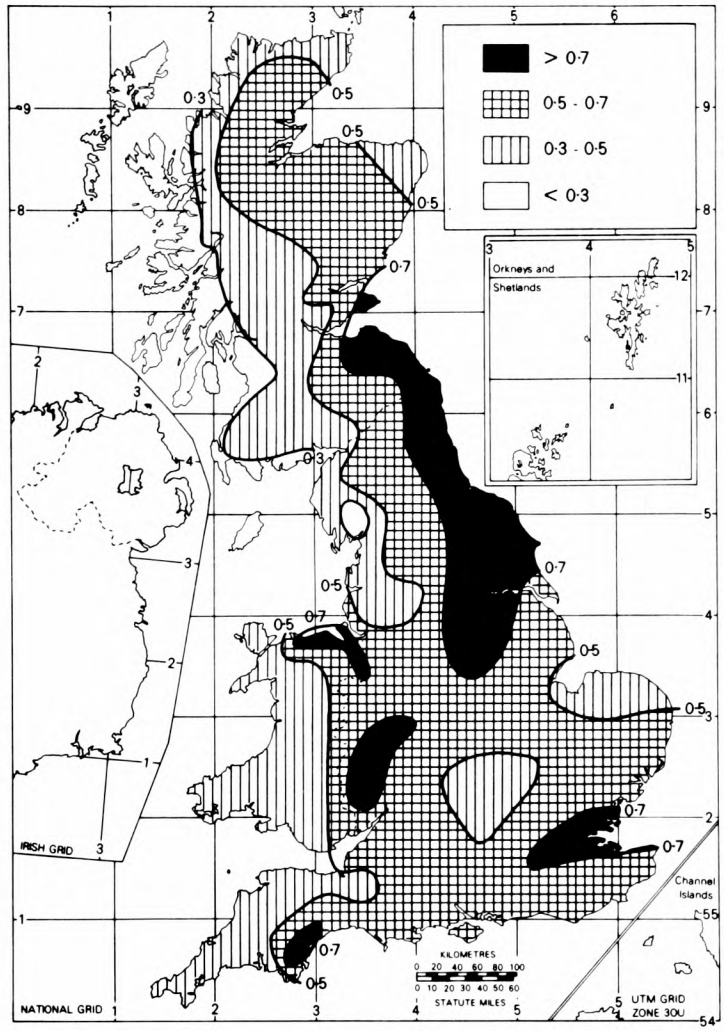

FIGURE 9. Coefficient of skewness of monthly rainfall from 1941 to 1970 (mean of all months)
Standard error ≈ 0.12.

In the above discussion n was taken as 1 (month). As the duration (n) is increased cases for analysis decrease and estimates of C_v, and hence of β_s become progressively less reliable. However, for large n it is expected that C_v will still be independent of $\bar{R}$ and hence it is assumed that $\beta_s \neq \beta_s(n)$. This is consistent with the finding that $\alpha \neq \alpha(\bar{R})$ intimated by equation (13).

5.3. *Estimation of* β_t

β_t measures the variation of C_v with $\bar{R}$, from month to month, at a place. Here the individual monthly values of C_v have to be considered separately and the errors involved are $\sqrt{12}$ times larger than when C_v can be meaned over all months. To obtain results with

small errors the three long sets of monthly totals were used to examine the variation of C_v with mean monthly rainfall. Figure 4 shows the seasonal variation of C_v of monthly rainfall. Variability is at a maximum in spring and a minimum in autumn. When C_v is plotted against mean monthly rainfall (Figure 1) higher variability is seen to be associated with lower mean rainfall.

This relationship may be incidental. Seasonal differences in the average general circulation of the atmosphere might be responsible for the spring peak in variability. The possible association of blocked westerlies with high variation in point rainfall was examined. The progressive index (P) of the *PSCM* indices first introduced by Murray and Lewis (1966) was taken as a rough measure of the degree of blocking and this was plotted against C_v of monthly rainfall: no relationship was revealed. This suggests that any trends shown in Figure 10 are due to the dependence of C_v upon $\bar{R}$.

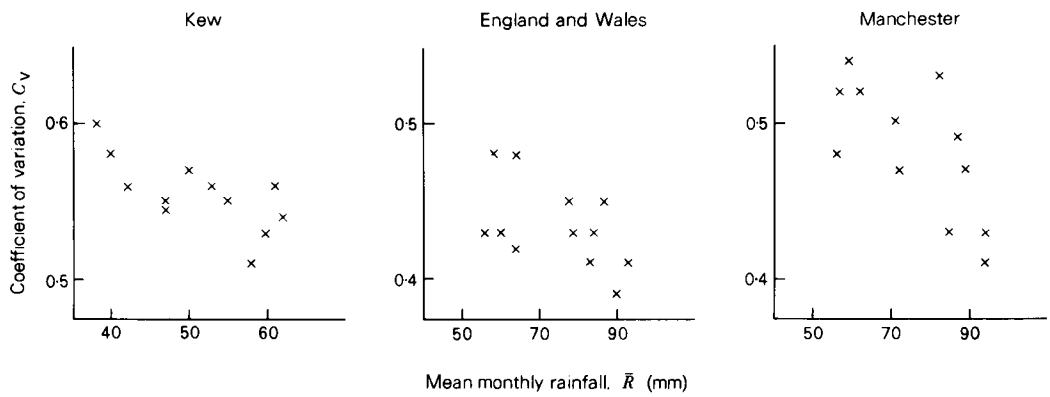

FIGURE 10. Coefficient of variation of monthly rainfall against mean monthly rainfall

From section 4.2, any relationship between C_v and $\bar{R}$ is assumed to take the form

$$C_v \propto \frac{1}{\bar{R}^{\beta_t}}$$

Values of β_t and standard errors (S.E.) for the three very long series are as follows:

Kew	$\beta_t = 0.14$	S.E. = 0.06
Manchester	0.25	0.11
England & Wales	0.17	0.09

Values of β_t were also calculated for the 96 stations (1911–70).

Random geographical variations gave values from −0.08 to 0.42 about a mean of 0.17. For β_t for durations of up to 6 months it was found that, to a first approximation

$$\beta_t = 0.17\exp\left\{-0.7(n-1)\right\}. \qquad \ldots(17)$$

Strictly, the previous findings $\alpha = 0.48 \neq \alpha(R)$ and $\beta_s = 0 \neq \beta(n)$ require there be no seasonal variation of C_v. Figure 4 shows that there is a very slight seasonal variation of

C_v and this results in the non-zero value of β_t given by equation (17). The magnitude of this seasonal variation, however, is too small to upset the results obtained for α and β_s, but still sufficiently well defined to warrant its acceptance.

5.4. *Rainfall distribution using C_v and C_s as input parameters*

The arguments presented above suggest that, to a first approximation, C_v over n months can be obtained from C_v for all single months by the version of equation (4) given in 5.0,

$$C_{v(n,R)} = \frac{C_{v(1,r)}}{n^{0.48}} \cdot \left(\frac{nr}{\bar{R}}\right)^{\beta t},$$

where β_t is given by equation (17). Following the assumption made at the end of 4.4 it is assumed that a similar equation applies to C_s.

To calculate rainfall/return-period relationships using C_v and C_s it is necessary to find a suitable distribution function for which these parameters can be used as input. In an interesting series of papers Stidd (1968, 1970, 1973) proposed that the cube roots of rainfalls are normally distributed. This is based on the assumption that rainfall amount is related to the product of three independent, normally distributed, physical variables. These are the vertical component of air motion, the depth of atmosphere being lifted at a given rate and the duration of the vertical motions.

If rainfall summed over any given duration did follow a cube-root normal distribution then C_s would be directly proportional to C_v. If C_v decreased according to the square root of the duration C_s would follow suit. Stidd postulated, however, that C_v should decrease with duration according to the cube-root relationship. The present author found this distribution inadequate for three reasons:

(a) Although it gave an adequate fit for rainfall summed over a number of months the fit for one month's rainfall was poor.

(b) The ratio of C_s to C_v required by the distribution was greater than that found in present study (1.67 as opposed to about 1.2); and also the distribution does not allow the ratio (of C_s to C_v) to vary from one location to another.

(c) His proposal that C_v should decrease with duration according to a cube-root relationship is in conflict with the approximately square root relationship found in the present investigation.

The square-root relationship receives support from the work of Mooley (1971). Investigating monsoon rainfall he found that the variance of rainfall over n months can be obtained by adding the variances of the rainfalls for the individual months. If there are no seasonal variations in C_v this implies that C_v decreases with duration according to $\sqrt{n}$.

A 3-parameter gamma distribution was fitted to the Kew data using the observed values of C_v and C_s for 1 month. A good fit was obtained except for falls of less than 10 per cent of average. A 3-parameter log-normal distribution was also fitted. The results were almost identical with those from the gamma distribution except that the fit at the dry end of the spectrum was slightly worse. A normal distribution adjusted for variation of skewness and kurtosis was also fitted. The results were inferior to those from the other two distributions. The choice between adjusted versions of the first two distributions fell on the log-normal, because the gamma distribution, having parameters which tend to

infinity as the skewness tends to zero, is not suited to near-normal distributions. Details of the log-normal distribution and of the adjusting factor applied are given in the Appendix.

5.5. *Comparison with 'observed' results*

Comparisons between the modelled and 'observed' distributions are presented in Tables III–V. The 'observed' rainfalls were obtained using the procedure described in section 3, where it was shown that ranking only 60 observed rainfalls into ascending order does not yield stable rainfall/return-period relationships. The merits of a modelled distribution must be tested against the largest and most reliable rainfall series available. In this comparison the Kew and Manchester series were used, the results being meaned over all months where possible. Table III compares modelled and 'observed' distributions of rainfall at Kew and Manchester, meaned over all starting months, for durations of 1 month and 12 months. There is generally good agreement of amounts (expressed as percentage of average rainfall) especially for return periods of less than 100 years. Table IV compares calendar month rainfalls for given return periods as modelled, as estimated by Wales-Smith (1973) and as given by the 'observed' method. The agreement is quite good; the largest discrepancies between the modelled and 'observed' results occurring in April, July, August and December.

Table V extends the same comparison to durations of 2 to 6 calendar months starting in October and April. Agreement is generally good, the largest differences occurring in the periods April to August and April to September.

TABLE III. *Percentage of average rainfall corresponding to given return periods, for duration of 1 month or 12 months*

Return period	Kew 1697–1973					Manchester 1786–1971			
	1 month			12 months		1 month		12 months	
years	A	B	C	B	C	B	C	B	C
200	284.2	284.2	286.4	147.1	152.6	256.4	263.7	141.1	144.3
100	260.0	260.0	258.0	142.1	143.4	236.8	244.2	136.8	137.5
50	235.2	235.2	236.6	136.7	138.4	216.5	219.2	132.1	132.2
20	201.0	201.0	204.0	128.8	129.9	188.1	189.5	125.3	125.2
10	173.3	173.3	175.4	121.9	122.8	164.8	164.8	119.4	119.1
5	143.0	143.0	142.4	113.9	114.0	138.8	138.9	112.4	112.0
−2	93.4	93.4	92.8	99.4	99.4	95.1	94.8	99.5	99.1
−5	53.0	53.0	52.7	85.7	85.5	58.3	58.3	87.3	87.6
−10	35.1	35.1	34.8	78.9	78.9	41.4	40.4	81.2	81.8
−20	22.0	22.4	23.6	73.5	72.9	28.7	27.8	76.3	77.1
−50	7.9	12.4	12.3	67.5	66.4	17.3	16.1	70.9	71.3
−100	0.0	7.9	8.1	63.7	60.3	11.8	13.3	67.4	66.8
−200	0.0	5.1	5.6	60.2	56.5	8.0	11.3	64.2	63.2

The Table is based on data for Kew (1697–73) and Manchester (1786–1971); comparison between log-normal distribution (A), corrected log-normal distribution (B) and the 'observed' rainfalls (C). The figures are meaned over all starting months. Positive return periods represent excesses, negative return periods represent deficiencies.

TABLE IV. *Monthly rainfalls corresponding to given return periods based on data for Kew from 1697 to 1973*

Return	Jan.			Feb.			Mar.			Apr.			May			June		
period	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C
years									*millimetres*									
100	122	123	120	108	104	104	105	103	100	102	111	98	116	122	115	139	132	135
50	109	111	109	98	94	95	98	93	97	94	100	93	106	111	106	122	119	119
20	94	94	94	81	80	81	84	79	82	82	85	83	95	94	96	103	102	103
10	82	81	82	71	68	70	70	68	68	72	73	73	81	81	82	86	88	91
5	66	67	67	58	56	56	56	56	55	59	60	60	64	67	64	72	73	74
−5	22	24	22	20	20	19	18	19	17	21	21	20	26	24	27	26	27	26
−10	15	16	15	12	13	12	11	12	10	12	14	13	17	16	18	18	18	18
−20	10	10	11	7	8	7	6	8	7	9	9	9	11	10	12	13	11	12
−50	6	6	6	3	4	4	2	4	3	4	5	4	8	6	8	7	6	7
−100	4	4	5	2	2	2	1	2	2	2	3	2	5	3	6	4	4	6

Return	July			Aug.			Sept.			Oct.			Nov.			Dec.		
period	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C
years									*millimetres*									
100	168	153	164	138	145	134	137	139	136	152	154	151	157	149	157	149	135	146
50	145	139	141	126	131	124	129	126	127	145	140	147	141	135	137	134	122	132
20	123	119	125	111	112	109	111	108	110	131	120	125	116	116	116	110	104	107
10	111	103	108	98	97	96	96	99	94	107	104	104	103	100	100	92	90	90
5	89	85	88	82	80	80	79	77	77	89	86	86	82	83	82	74	74	73
−5	30	33	30	30	30	29	30	29	29	34	33	34	33	32	32	28	28	26
−10	22	22	22	18	20	18	18	19	17	24	23	23	23	21	25	17	18	16
−20	14	15	16	12	13	12	10	12	10	15	15	16	16	14	17	12	12	12
−50	8	8	10	5	7	3	5	7	3	7	8	8	11	8	15	6	7	9
−100	4	5	4	2	5	2	2	4	2	4	5	4	9	5	10	3	4	3

Comparison between the results of Wales-Smith (A), the present work (B) and the 'observed' rainfalls (C). Positive return periods refer to excesses; negative return periods refer to deficiencies.

5.6. *Confidence limits*

Four input variables are used, C_v, C_s, α and β_t, the last being less important than the other three, since it only introduces small differences between starting months for durations of less than about 4 months. The four parameters are used to provide results expressed as fractions of average rainfall. To obtain amounts in absolute units $\bar{R}$ must be known. To gain some idea of the confidence limits of the modelled results each of the parameters $\bar{R}$, C_v, C_s and α was varied by 2 standard errors. The calculations were made for a dummy station with $\bar{R}_A$ 1000 mm and for which the average values for England and Wales $C_v = 0.53$ and $C_s = 0.63$ were assumed. Standard errors, based on an assumed 60-year record length were:

$\bar{R}_A$ 2 per cent,
C_v 0.01,
C_s 0.09,

TABLE V. *Rainfalls for durations of 2 to 6 months, starting in October and April, and corresponding to given return periods*

Return period	Oct.–Nov.			Oct.–Dec.			Oct.–Jan.			Oct.–Feb.			Oct.–Mar.		
	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C
years	*millimetres*														
100	239	247	237	307	319	306	371	380	372	425	427	422	477	473	476
50	229	228	229	297	299	292	360	358	358	399	404	398	462	449	457
20	201	202	203	273	269	268	328	326	329	374	371	379	425	415	424
10	178	181	186	244	245	249	304	299	305	345	343	354	392	385	398
5	157	157	156	216	216	219	267	268	271	314	310	316	355	352	358
−5	83	79	80	127	123	125	169	165	166	198	201	199	234	236	234
−10	63	63	61	106	103	102	143	142	139	177	175	169	209	209	203
−20	52	50	49	89	87	86	125	123	120	155	155	149	189	188	182
−50	38	36	39	72	70	73	105	103	110	133	134	130	165	165	165
−100	31	29	31	58	59	67	93	91	89	120	120	118	151	150	155
−1000	18	13		42	33		67	58		90	84		116	111	

Return period	Apr.–May			Apr.–June			Apr.–July			Apr.–Aug.			Apr.–Sept.		
	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C
years	*millimetres*														
100	179	185	177	275	261	272	369	348	363	490	426	495	550	497	527
50	163	175	163	255	244	258	346	328	344	450	403	438	510	472	515
20	147	152	148	222	220	223	313	299	313	394	370	389	458	436	464
10	134	135	133	199	199	196	282	274	276	353	342	347	417	405	425
5	116	117	116	175	176	177	244	245	248	311	310	311	370	370	370
−5	58	58	58	100	100	97	151	151	149	197	200	191	240	248	241
−10	47	45	47	81	83	83	130	129	128	173	175	171	215	220	215
−20	36	36	37	69	70	66	115	112	109	155	155	154	195	197	191
−50	27	26	29	58	56	58	98	94	89	136	134	138	174	173	173
−100	22	20	19	53	47	56	88	83	80	125	120	132	159	157	153
−1000	16	9		41	27		67	53		97	84		127	116	

The Table is based on data for Kew from 1697 to 1973. Comparison between the results of Wales-Smith (A), the present work (B) and the 'observed' rainfalls (C). Positive return periods refer to excesses; negative return periods refer to deficiencies.

The standard error of α is difficult to obtain. The sensitivity of results to variation of α is indicated by allowing it to vary from 0.46 to 0.50.

Modelled rainfall distributions for C_v duration of 12 months are presented in Figure 11. It can be seen that the results are sensitive to $\bar{R}_A$, α, C_v and C_s in that order. If $\bar{R}_A$ is varied by ±2 standard errors the rainfall amount associated with a given return period varies by ± 4 per cent but the return period corresponding to a given rainfall amount may vary between about $0.5T$ and $2T$. As an example, Figure 11 shows that the annual rainfall which corresponds to a 'dry' return period of 50 years is 685 mm but that, as a result of uncertainties in the average annual rainfall, this value may lie between 658 mm and

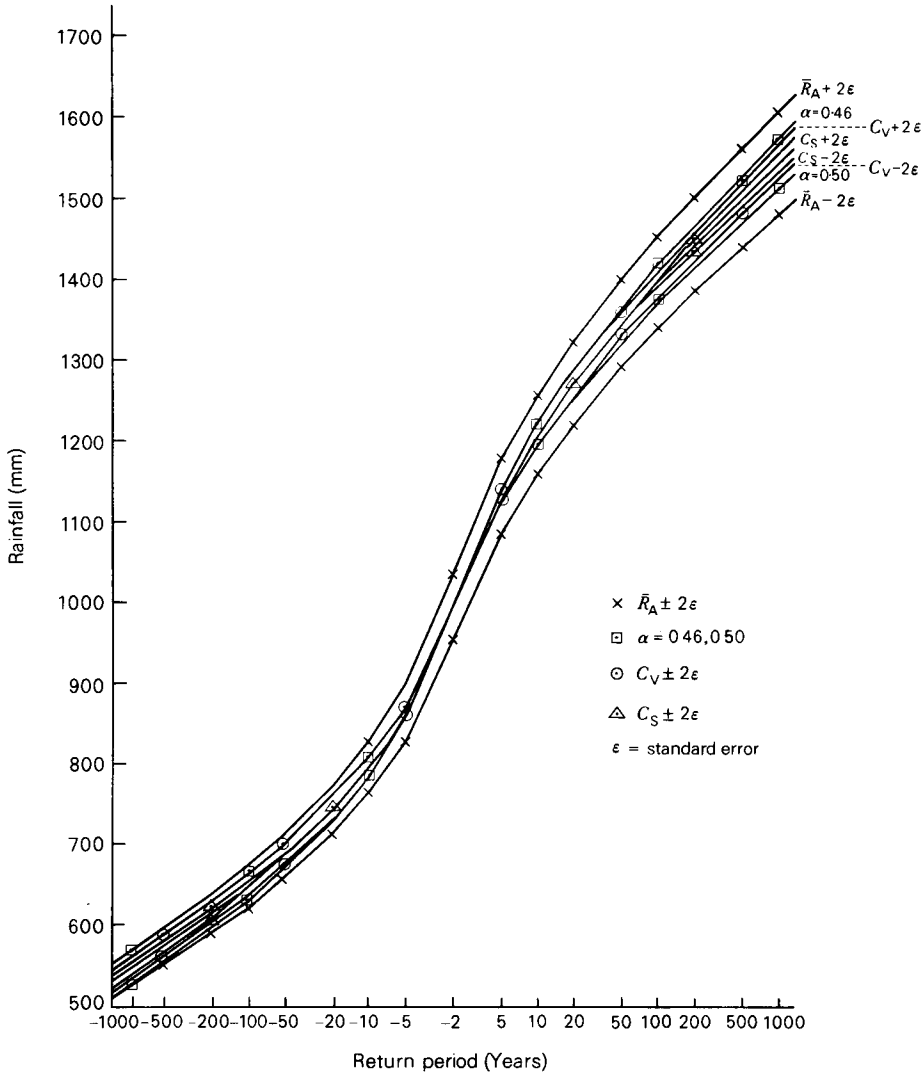

FIGURE 11. Effect on distribution of annual rainfall of varying $\bar{R}_A$, C_v, C_s and α by 2 standard errors

Negative values for return periods refer to deficiencies and positive values to excesses.

712 mm. Similarly, the return period of an annual rainfall of 685 mm may lie between about 30 years and 80 years.

Overall confidence limits, taking into account simultaneous errors in all input parameters, are difficult to evaluate rigorously because the correlations between the errors are unknown. In general, however, the overall confidence limits should not differ greatly from those due to $\bar{R}_A$.

6. THE VARIABILITY OF AREAL RAINFALL

Areal reduction factors of C_v and C_s may be defined as

$$D_v = \frac{C_{v,A}}{C_{v,P}} = \exp(-\gamma_v A)$$

and

$$D_s = \frac{C_{s,A}}{C_{s,P}} = \exp(-\gamma_s A)$$

where γ_v and γ_s are defined as the areal reduction coefficients of C_v and C_s respectively. It follows that

$$\gamma_v = \frac{\ln(C_{v,P}) - \ln(C_{v,A})}{A}$$

with a similar expression for γ_s. The standard error of γ_v can be estimated from the standard errors of $C_{v,P}$ and $C_{v,A}$ if these variables are independent of one another, but this latter condition is not fulfilled. In the following text, therefore, only vague references to the limits of accuracy are made, these being based mainly on the scatter of results obtained. It is important to remember that, where results can be meaned over all months, the smallest errors are associated with the shortest durations examined (i.e. 1 month).

6.1. *Calculation of areal reduction factors of C_v and C_s*

Areal rainfalls used in the calculations of D_v and D_s were derived as follows:

(a) The monthly rainfall totals were expressed as fractions of the average annual rainfall, $\bar{R}_A$, at each station within the area.

(b) The monthly areal rainfall, expressed as a fraction of areal $\bar{R}_A$, was obtained by meaning the fraction of station $\bar{R}_A$ over all stations within the area (X).

(c) The areal $\bar{R}_A$ was obtained by meaning the $\bar{R}_A$ over all stations within the area (Y).

(d) The monthly areal rainfall, expressed in absolute units, was obtained from the products of X and Y. D_v and D_s were then calculated as follows:

(1) C_v and C_s of the individual station records were calculated and then meaned over all stations within the area. These mean values were used as estimates of $C_{v,P}$ and $C_{s,P}$.

(2) C_v and C_s of the areal rainfall series were calculated and used as estimates of $C_{v,A}$ and $C_{s,A}$.

(3) The ratios of $C_{v,A}$ to $C_{v,P}$ and $C_{s,A}$ to $C_{s,P}$ were taken as the values of D_v and D_s respectively.

6.2. *Effect on estimates of C_v and C_s of the number of stations used*

Before investigating the variation of C_v and C_s with area, it is necessary to ascertain the number of stations required within a given area to obtain sufficiently accurate values of

C_v and C_s. The C_v and C_s of monthly and annual rainfall over England and Wales during the period 1911–70 were calculated using a variable number of rainfall stations. The figures for C_s were too unstable to provide much information, but values of C_v are presented in Table VI. The results show, as expected, that C_v is greater for a point than for an area but, instead of a progressive decrease with an increasing number of stations, there is a rapid decrease from a single station to a small number of stations followed by a slow *increase* as the number of stations is increased further. The results show that C_v depends mainly upon the mean distance of stations from the centre of the area. The use of four stations which are located at the extremities of the area will give rainfalls which are more diverse than those derived from a greater number of stations located closer to one another. In general, however, it may be said that C_v is insensitive to the number of stations used. The present investigation allows for 48 stations over an area the size of England and Wales, and Table VI shows that this is ample for obtaining sufficiently accurate values of C_v.

TABLE VI. *Effect of number of stations used on the calculation of C_v of rainfall over England and Wales*

Parameter	Number of stations							
	1	4	12	24	34	48	68	96
C_v over 1 month	0.546	0.408	0.422	0.419	0.423	0.422	0.421	0.423
C_v over 12 months	0.161	0.118	0.124	0.125	0.126	0.126	0.126	0.127

The calculated values of C_v are for the period 1911–70 and are meaned over all starting months.

6.3. *Observed relationships of C_v and C_s with duration, mean rainfall and area*

Of the expressions (5) to (12) developed in subsection 4.4, relations (8) to (12) remain to be investigated. To examine the variation of C_v and C_s with area, seven networks of stations were devised. Starting with points at Kew, Batheaston, Bolton and Birdsall, progressively larger areas were used until they each covered one quarter of the area of England and Wales. Four of the networks used are displayed in Figure 12.

Values of D_v and D_s were calculated for durations ranging from one month to 120 months and starting in each month of the year. Some values of D_v, meaned over all starting months, are presented in Table VII for a small selection of duration periods. Values of D_s are not quoted because the standard errors are too large to permit meaningful figures to be obtained. This situation was commonly encountered and so, backed by first principles, the true behaviour of C_s is assumed to be similar to that of C_v.

Table VII presents values of D_v for 28 different areas, and in all of these D_v is seen to be essentially independent of duration. Any apparent changes are in the opposite sense to those which were expected, but are within the limits of error. It is therefore concluded that, for the duration of 1 month and more, D_v and D_s are independent of duration, that is to say

$$\gamma_v \neq \gamma_v(n) \text{ and } \gamma_s \neq \gamma_s(n). \qquad \ldots (8)$$

TABLE VII. *Areal reduction factors of the coefficient of variation of rainfall, meaned over all starting months, for the period 1911–70*

Areas in south-east of England and Wales (centred on Kew)

Area (100 km²)	Duration (months) 1	3	6	12
1.8	0.973	0.972	0.975	0.976
4.1	0.968	0.967	0.964	0.961
17	0.949	0.947	0.946	0.944
29	0.943	0.943	0.942	0.940
70	0.927	0.929	0.931	0.927
141	0.910	0.910	0.912	0.914
338	0.890	0.891	0.894	0.891

Areas in south-west of England and Wales (centred on Batheaston)

Area (100 km²)	Duration (months) 1	3	6	12
2.1	0.977	0.972	0.970	0.974
4.4	0.965	0.962	0.960	0.960
13	0.949	0.947	0.942	0.944
29	0.942	0.939	0.935	0.932
72	0.933	0.928	0.923	0.923
115	0.909	0.901	0.895	0.891
350	0.899	0.893	0.888	0.882

Areas in north-west of England and Wales (centred on Bolton)

Area (100 km²)	Duration (months) 1	3	6	12
2.3	0.969	0.963	0.959	0.953
4.8	0.958	0.954	0.946	0.935
15	0.946	0.944	0.934	0.927
30	0.932	0.928	0.924	0.914
69	0.912	0.910	0.900	0.894
157	0.877	0.870	0.860	0.849
343	0.856	0.851	0.833	0.817

Areas in north-east of England and Wales (centred on Birdsall)

Area (100 km²)	Duration (months) 1	3	6	12
2.4	0.979	0.969	0.968	0.963
5.7	0.959	0.953	0.952	0.947
15	0.949	0.946	0.945	0.938
33	0.931	0.929	0.926	0.922
65	0.912	0.908	0.911	0.901
149	0.884	0.887	0.892	0.884
353	0.854	0.857	0.862	0.854

This conclusion receives some support from the work of Huff and Shipp (1969), who found that the decrease with distance of the correlation of seasonal rainfall was similar to that of monthly rainfall.

A corollary to the above conclusion (see equation 15) is that the decrease of C_v with duration should be the same for the areas as for points, that is

$$\alpha \neq \alpha\,(A). \qquad \ldots (9)$$

A constant value of α (of approximately 0.5), independent of area, is consistent with the fact that consecutive monthly rainfall totals are independent for areas as well as for points.

By considering each month separately it is possible to see how the dependence of C_v upon $\bar{R}$ at a given location varies with area. In Table VIII, values of β_t for an area, denoted by $\beta_{t,A}$, are quoted alongside joint values of β_t (meaned over all stations within an area) and denoted by $\beta_{t,P}$. The errors associated with each calculation are large, but there is no evidence that the ratio $\beta_{t,A}/\beta_{t,P}$ decreases as the area increases. It is therefore concluded that

$$\beta_t \neq \beta_t\,(A). \qquad \ldots (10)$$

TABLE VIII. *Variation of C_v of monthly rainfall with mean rainfall ($\bar{R}$) for the period 1911–70 by transition from a dry to a wet month as represented by β_t in $C_v \propto 1/\bar{R}^{\beta_t}$*

Approx. area (100 km²)	Areas of England and Wales in: South-east		South-west		North-west		North-east	
	$\beta_{t,P}$	$\beta_{t,A}$	$\beta_{t,P}$	$\beta_{t,A}$	$\beta_{t,P}$	$\beta_{t,A}$	$\beta_{t,P}$	$\beta_{t,A}$
2	0.34	0.36	0.18	0.16	0.13	0.10	0.35	0.37
5	0.32	0.32	0.23	0.21	0.14	0.09	0.34	0.37
15	0.26	0.23	0.23	0.20	0.21	0.17	0.33	0.36
30	0.29	0.28	0.20	0.16	0.20	0.17	0.31	0.34
70	0.26	0.23	0.20	0.16	0.16	0.13	0.31	0.37
150	0.22	0.17	0.19	0.14	0.19	0.18	0.32	0.44
340	0.25	0.25	0.17	0.10	0.21	0.16	0.31	0.42

$\beta_{t,P}$ and $\beta_{t,A}$ denote values obtained from point and areal rainfall respectively.

When $\bar{R}$ is changed by passing from a dry to a wet location, the true variation of C_v with $\bar{R}$ is complicated by the geographical variations. In subsection 5.2 it was concluded that $\beta_s = 0$. In a separate investigation for areas, it would be difficult to distinguish between variations of C_v which were due to a change of $\bar{R}$ and those which are due to a change of location. If C_v is independent of $\bar{R}$ for points, it is difficult to find any reason why C_v should not be independent of $\bar{R}$ for areas. It is therefore assumed that $\beta_s = 0$ for both points and areas and that

$$\beta_s \neq \beta_s(A). \qquad \ldots(11)$$

In determining any variation of γ_v with $\bar{R}$, geographical variation in γ_v would again mask any genuine relationship with $\bar{R}$. From equation 14

$$\beta_s \neq \beta_s(A) \text{ and } \beta_t \neq \beta_t(A) \text{ implies } \gamma_v \neq \gamma_v(\bar{R}).$$

It is therefore assumed that

$$\gamma_v \neq \gamma_v(\bar{R}). \qquad \ldots(12)$$

6.4. *Effect of the shape of an area*

The shape of an area will affect the variability of the rainfall measured over the whole area. The correlation of one point rainfall with another may be expected to be related to the distance between them. The more elongated the shape of an area, the greater the mean distance of points from the centre of the area, and hence the greater the decrease in variability. It seems reasonable therefore that a suitable measure of an area would be the mean distance of uniformly distributed stations from the centre of the area (d). This agrees with the results of subsection 6.2, where variations in C_v were interpreted as being due to changes in the mean distance, d.

In order to assess the merit of the parameter d, networks of stations were devised in which the magnitude of the area was held constant while the shape was varied. Two of

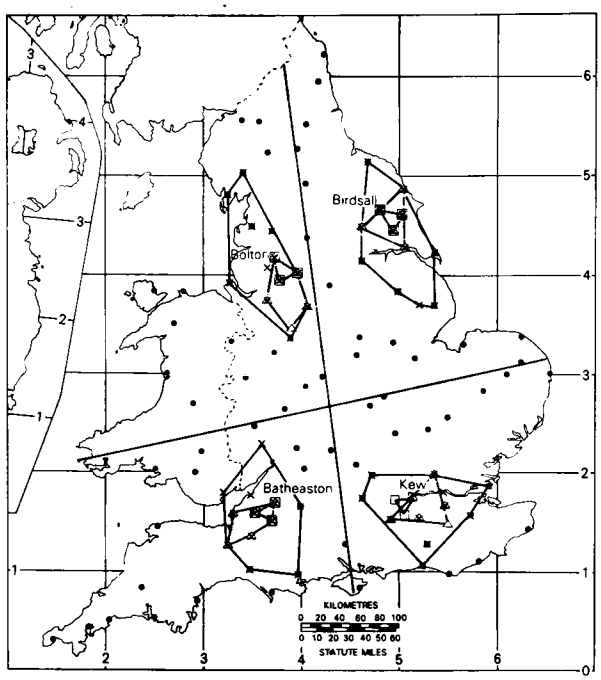

FIGURE 12. Station networks used to investigate the variation of C_v with area

□ Network 1 (3 stations);
△ Network 3 (6 stations);
x Network 5 (12 stations);
● Network 7 (24 stations).

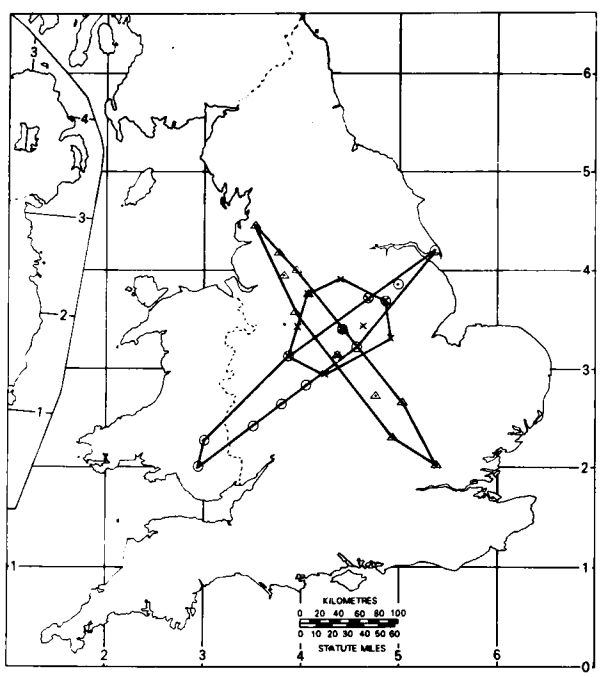

FIGURE 13. Station networks used to investigate the variation of C_v with the shape of an area

x Network 1;
o Network 4, SW – NE orientation;
△ Network 4, NW – SE orientation.

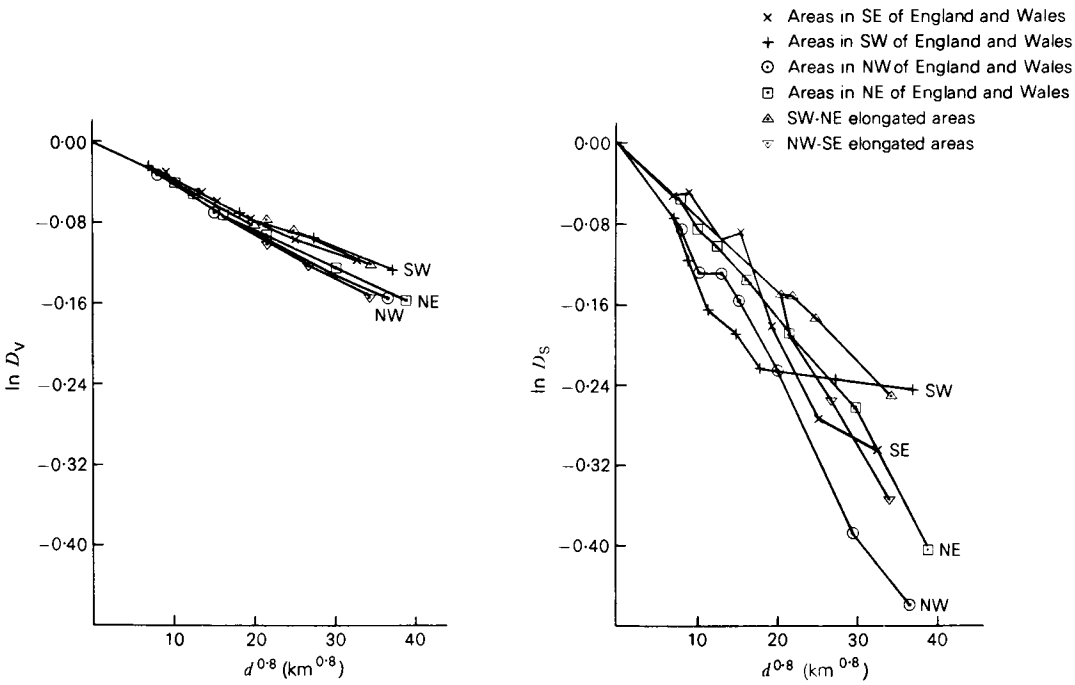

FIGURE 14. Log of the areal reduction factors of C_v and C_s against $d^{0.8}$ for a duration of one month (and meaned over all months) for the period 1911–70

Comparison between reduction factors of (a) coefficient of variation, and (b) coefficient of skewness.

these networks, which use records for the period 1911–70, are displayed in Figure 13. Starting from a reasonably circular area, the networks were elongated in south-west to north-east and north-west to south-east directions, the two alignments helping to identify any changes in rainfall correlation with direction which may have been present. Values of D_v and D_s from this group of networks were then compared with those from the network of stations shown in Figure 12, where the size of the area was varied for shapes which were not unduly elongated.

The values of D_v and D_s with the smallest errors are those for a duration of one month which are meaned over all months; these are presented in Figure 14. Linear relationships were obtained by plotting $\ln D_v$ and $\ln D_s$ as ordinates against $d^{0.8}$ as abscissa, implying a relationship of the form

$$D_v = \exp(-\gamma_v d^{0.8}). \qquad \ldots (18)$$

The finding that D_v is related to d^z rather than to d, with $0 < z < 1$, is supported by the fact that similar relationships have been found between spatial rainfall correlation and distance by many authors, including Huff and Shipp (1969) and Yevjevich and Karplus (1973).

Two main items of interest arise from Figure 14:

(a) The graphs for the elongated areas are in accord with those for the more circular areas. They show that the rainfall is more poorly correlated in a north-west to south-east direction than in a south-west to north-east direction. Preferred directions of rainfall correlation have been noted by many workers, for example Caffey (1965) and Hendrick and Comer (1970). This and other geographical variations apart, it is clear that d is a suitable measure of area for the present purposes.

(b) D_s falls off with increasing area more rapidly than D_v. This is because rainfall deficiencies are spatially more coherent than rainfall excesses are, so that when rainfall is averaged over an area, most of the reduction in variability which takes place will be due to a reduction of very high rainfall totals. By meaning results over all the areas examined, it was found that:

$$\gamma_v = 0.0040 \text{ and } \gamma_s = 0.010 \text{ so that } \gamma_s = 2.5\ \gamma_v. \qquad \ldots (19)$$

For a circular area, d is two-thirds of the radius of the circle.

6.5. *Seasonal variations in the areal reduction coefficient*

The networks of stations displayed in Figure 12 were used in investigations of seasonal variations of the areal reduction coefficient. Values of D_v were calculated for each calendar month and examined separately; these showed the considerable geographic variations as illustrated in Figure 15. Important features are:

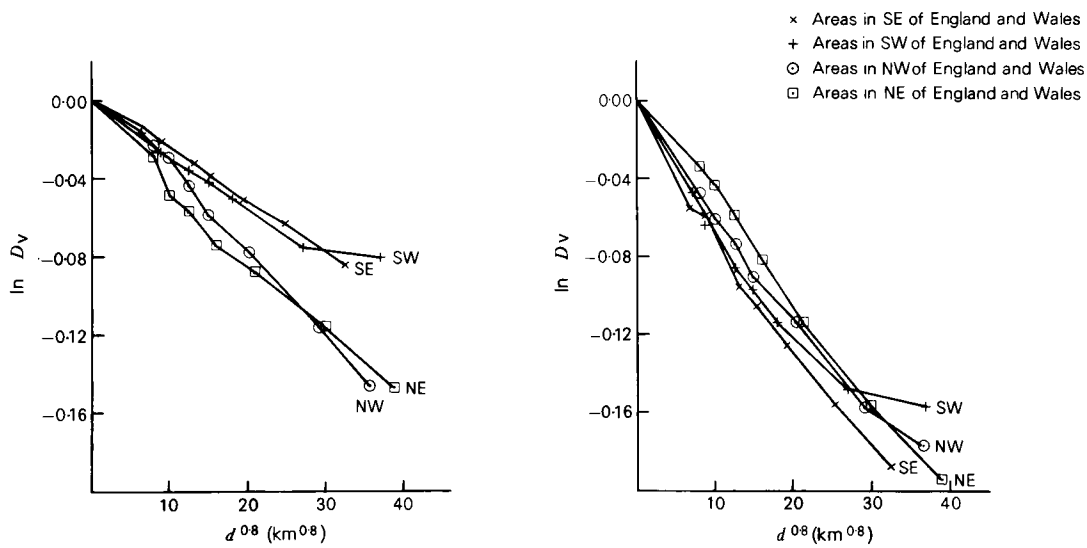

FIGURE 15. Log of the areal reduction factor of C_v against $d^{0.8}$ for a duration of one month for the period 1911–70

Comparisons between reduction factors (a) meaned over December, January and February, and (b) meaned over June, July and August.

(a) D_v falls off with increasing d more rapidly in summer than in winter; this is associated with the greater convective activity in summer than in winter. Huff and Shipp (1969) found that the decay of rainfall correlation with distance was greater in summer than in winter.

(b) Regional differences are less in summer than in winter. The geographical variations are due mainly to the effect of relief, and this is small during convective activity in the summer.

(c) In the relation $D_v = \exp(-\gamma_v d^z)$ – a generalized form of equation (18) – both γ_v and z vary with season. Empirically, however, this variation may be conveniently expressed by taking z as constant at 0.8 and allowing γ_v to vary freely.

Using equation (18), γ_v was calculated for each calendar month and the results are presented in Figure 16. There is a pronounced seasonal variation of γ_v in south-east England but very little in north-east England.

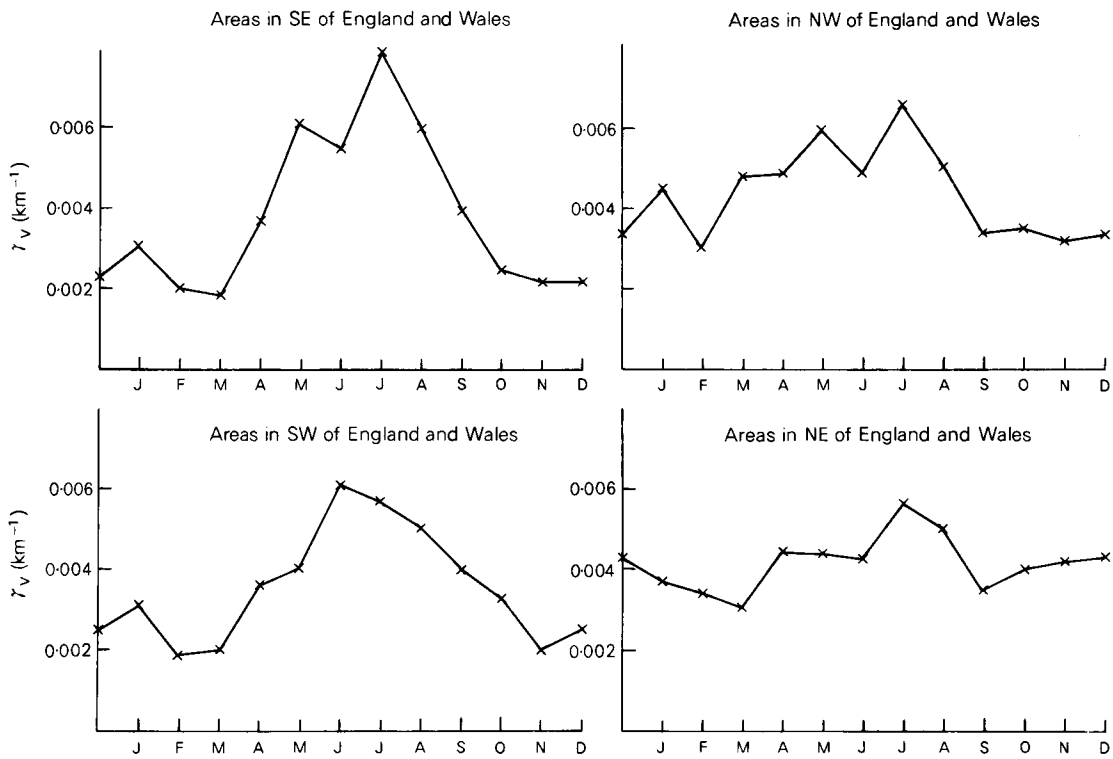

FIGURE 16. Seasonal variation of areal reduction coefficient of C_v for a duration of one month during the period 1911–70

6.6. *Spatial correlation of monthly rainfall*

To gain further information on the geographical variations of γ_v, the spatial correlation of monthly rainfall was investigated. Patterns of rainfall correlation have been examined by many workers, for example Glasspoole (1926), Caffey (1965) and Hendrick and Comer (1970). Figures 17 (a) to 17 (d) display the correlation coefficient between monthly

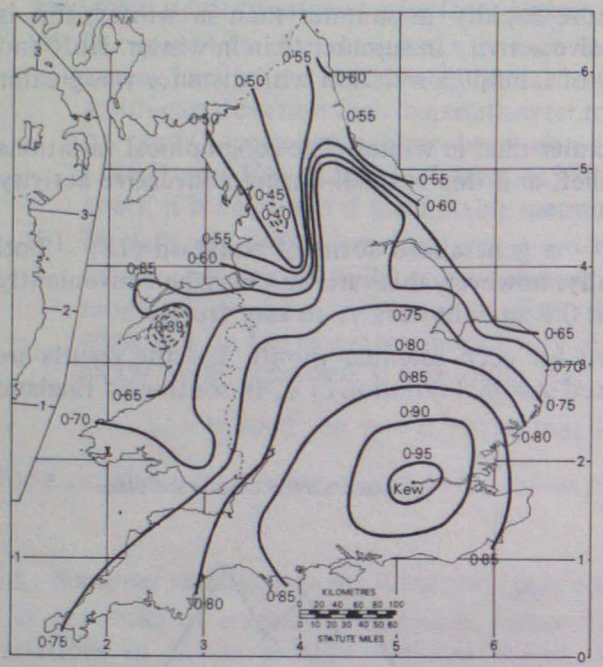

FIGURE 17a. Correlation coefficient of monthly rainfall with that at Kew for 1911–70

Mean of December, January and February.

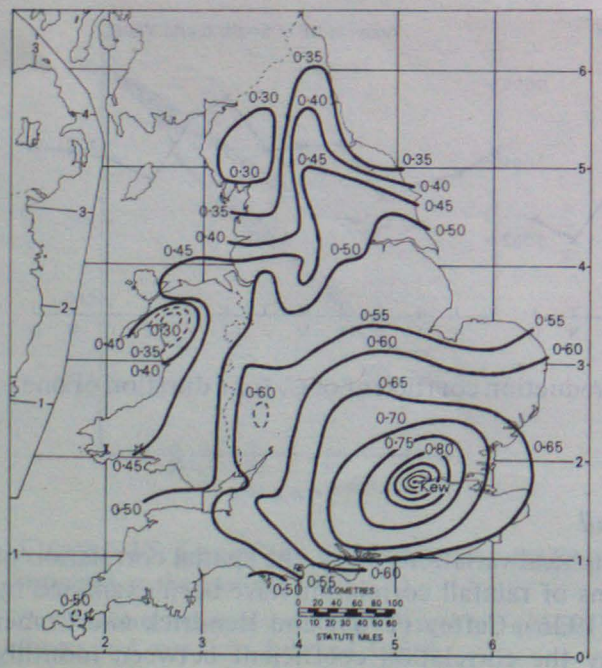

FIGURE 17b. Correlation coefficient of monthly rainfall with that at Kew for 1911–70

Mean of June, July and August.

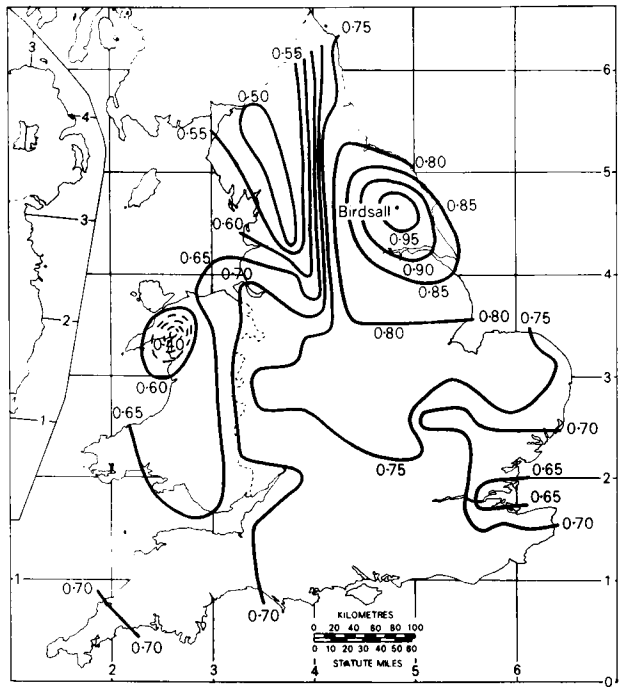

FIGURE 17c. Correlation coefficient of monthly rainfall with that at Birdsall for 1911–70

Mean of December, January and February.

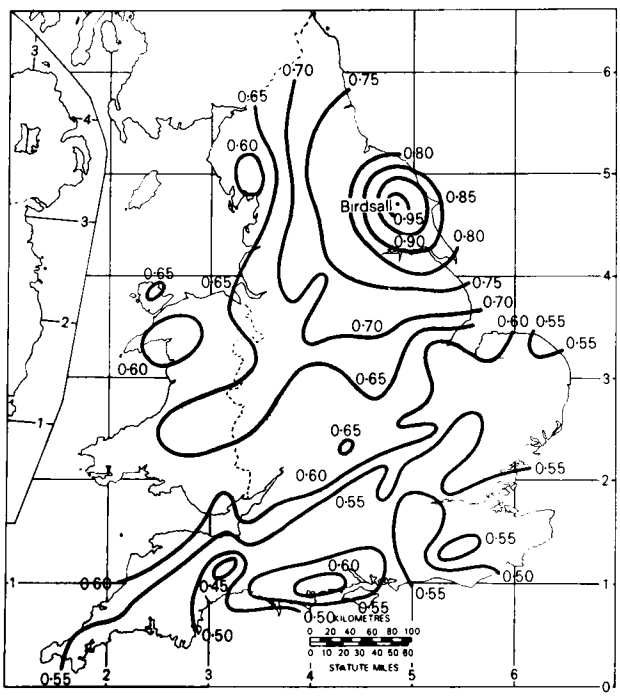

FIGURE 17d. Correlation coefficient of monthly rainfall with that at Birdsall for 1911–70

Mean of June, July and August.

rainfall at 130 points in England and Wales and at Kew and Birdsall. The maps reveal similar features to those observed for the areal reduction coefficient, namely:

(a) In areas of gentle relief the correlation falls off more rapidly in summer than in winter.
(b) The effect of relief on correlation is much greater in winter than in summer.
(c) In areas of gentle relief correlation falls off more steadily with distance in winter than in summer.
(d) Seasonal differences in correlation are more pronounced in the south-east than in the north-east.

The maps also demonstrate that the pattern cannot be inferred entirely by reference to the underlying relief. It follows that areal reduction coefficients cannot be obtained from an average value 'corrected for relief'. The value for a particular area can only be obtained by direct calculation.

6.7. *Geographical variations in the areal reduction coefficient*

Figure 17 clearly illustrates the roles of convection and relief in determining the areal reduction coefficient. Further investigations into geographical variations were made by using monthly rainfalls from a large number of stations over Great Britain for the period 1941–70. Groups of three stations were used to form about 240 small triangular areas for which d lay in the range 10–15 km and equation (18) was used to calculate γ_v for a duration of one month meaned over all months for the three winter months and for the three summer months. Space does not permit these maps to be reproduced here, but maps of γ_v over river catchments, displaying essentially the same phenomena, are presented later on. The summer values of γ_v displayed insignificant regional differences while winter values produced marked variations, the lowest values occurring on the north-east-facing slopes of eastern Scotland and north-east England. These findings are consistent with those of Goh and Lockwood (1974) who investigated the relationship between mean rainfall ($\bar{R}$) and altitude in the central Pennines. They found that the correlation between $\bar{R}$ and altitude was better on the eastern side of the Pennines than on the western side, and better in winter than in summer.

Figure 18 shows schematically a peninsular mountain ridge lying normal to the wind direction. Winds blowing from the sea to the hills are relatively wet, and those blowing in the reverse sense are relatively dry. Thus in Figure 18 points A and B will both be wet while point C will be dry. Rainfall correlation on the windward slopes of high ground will

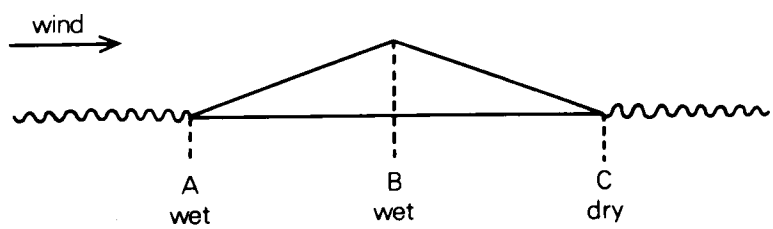

FIGURE 18. Effect of mountain ridge on spatial correlation of rainfall

therefore be much wetter than on the leeward slopes. The geographical and seasonal differences in γ_v are readily explained. In winter the effects of relief are strong, with low values of γ_v occuring on level ground while high values of γ_v are found on the prevailing leeward (north-eastward-facing) slopes of high ground.

6.8. *Representation of seasonal variations in the areal reduction coefficient*

The seasonal variation of the monthly values of γ_v is not a simple sinusoidal pattern, but comprises a broad winter base and a sharper summer maximum (or minimum). A simple scheme to represent this regime is shown in Figure 19. The months from October to March are given equal values of γ_v which form a broad base from which the values for other months are incremented by an amount h, a maximum (or minimum) being reached in June and July.

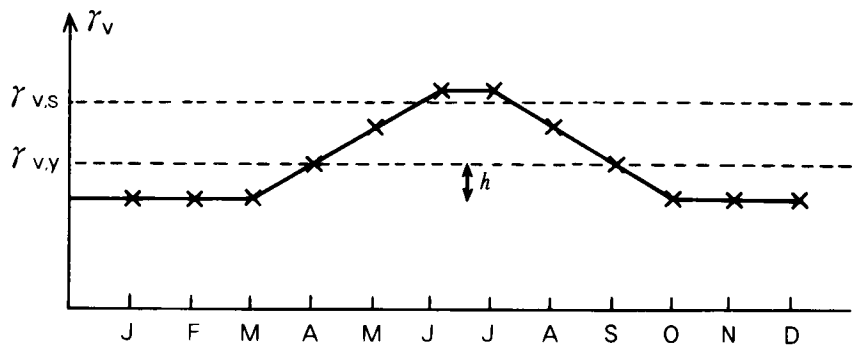

FIGURE 19. Scheme for representing seasonal variation in areal reduction coefficient

Since it was found that the areal reduction factor was independent of duration, all the calculations of γ_v have been made for a duration of one month. Seasonal differences in γ_v, however, cannot be expected to be independent of duration as, being mainly due to convection, they are most marked over short durations. Change of the seasonal variation of γ_v with duration was investigated using the station networks displayed in Figure 12 and records for the period 1911–70.

The following expressions were then written:

$\gamma_{v,1}$ = γ_v for a duration of one month (month m);

$\gamma_{v,n}$ = γ_v for a duration of n months centred on month m;

$\gamma_{v,y}$ = γ_v for a duration of one month, meaned over all months;

$\gamma_{v,s}$ = γ_v for a duration of one month, meaned over June, July and August;

it was found that (to a fair degree of approximation)

$$(\gamma_{v,n} - \gamma_{v,y}) = (\gamma_{v,1} - \gamma_{v,y}).\exp\{-0.16(n-1)\}.$$

The discussion on geographical and seasonal variations has so far been confined to C_v. The parameter C_s is too unstable to give reliable values of γ_s. The overall relationship from equation (19), i.e. $\gamma_s = 2.5\gamma_v$, is therefore used to obtain theoretical values of γ_s.

Values of the areal reduction coefficient for all months and all durations obtained from $\gamma_{v,y}$ and $\gamma_{v,s}$ are summarized as follows:

From Figure 19	$h = 0.6\,(\gamma_{v,s} - \gamma_{v,y})$	...(20)
From October to March	$\gamma_{v,1} = \gamma_{v,y} - h$	...(21)
From April and September	$\gamma_{v,1} = \gamma_{v,y}$	...(22)
For May and August	$\gamma_{v,1} = \gamma_{v,y} + h$	...(23)
For June and July	$\gamma_{v,1} = \gamma_{v,y} + 2h$	...(24)
For a duration of n months	$\gamma_{v,n} = \gamma_{v,y} + (\gamma_{v,1} - \gamma_{v,y})\exp\{-0.16(n-1)\}$	...(25)
In all cases	$\gamma_s = 2.5\,\gamma_v$.	...(19)

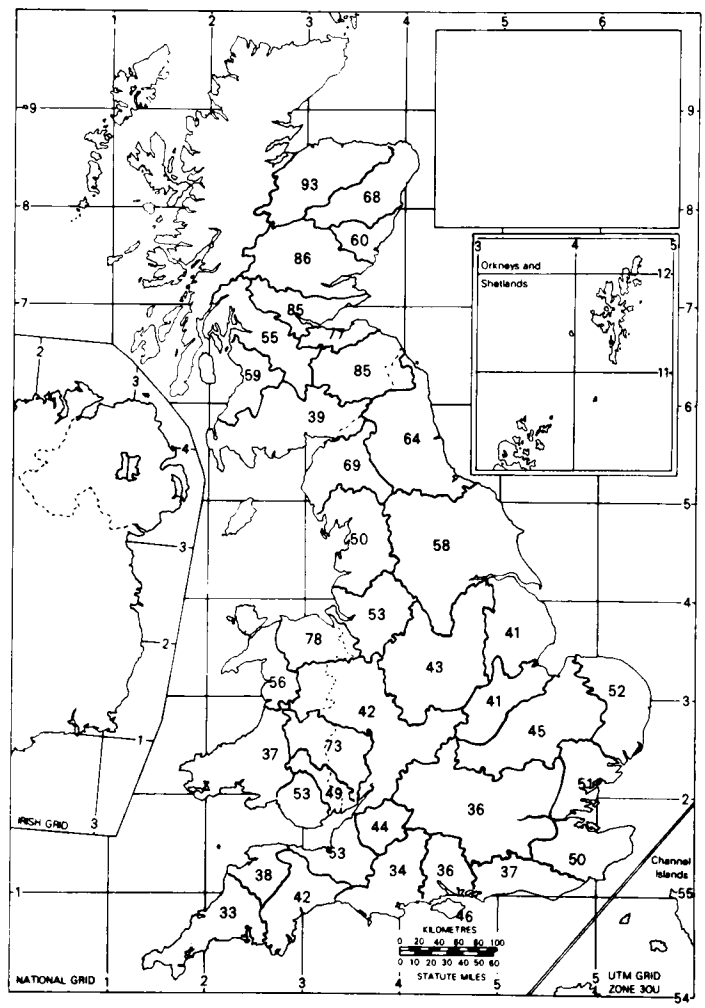

FIGURE 20. Areal reduction coefficient of variation of monthly rainfall for river catchments for 1941–70 i.e. γ_v in $C_{v,A} = C_{v,P}\exp(-\gamma_d\, d^{0.8})$.

(a) Mean of all months. Values quoted are $\gamma_v \times 10^4$ for d in km.

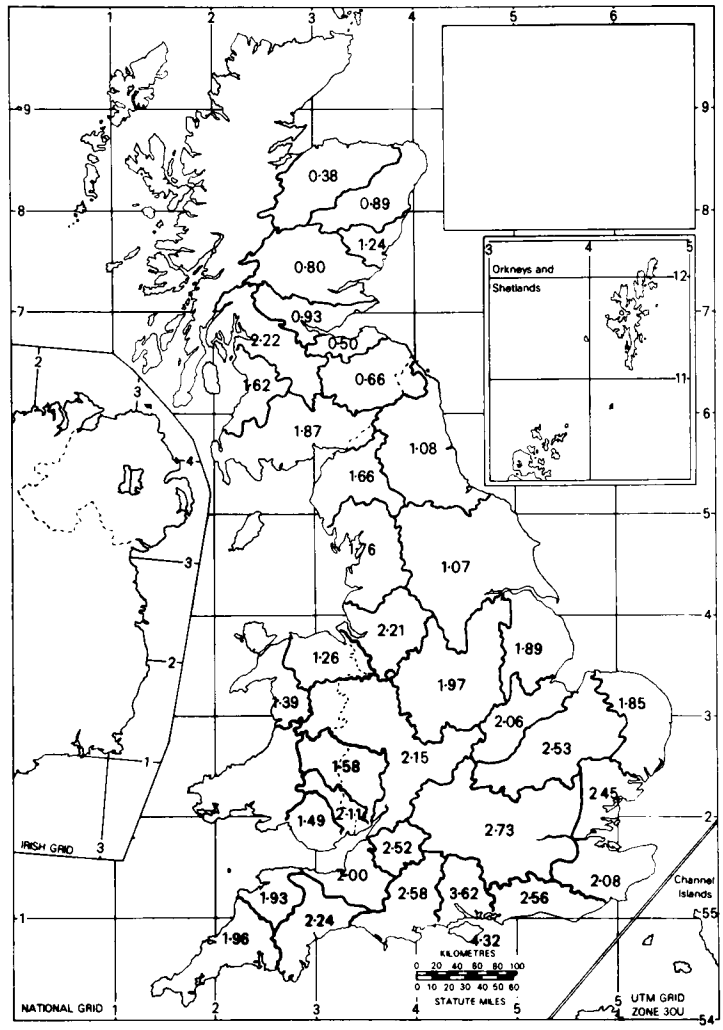

FIGURE 20—*continued.*

(b) Ratio of summer (mean of June, July and August) to winter (mean of December, January and February) values.

6.9. *Areal reduction coefficients for river catchments*

Calculations of γ_v were made for the areas covered by the River Purification Boards of Scotland and the River Divisions of the Regional Water Authorities of England and Wales. The calculations were made for the period 1941–70 using networks of stations uniformly distributed in each catchment. Figure 20(a) presents values of γ_v for one month, averaged over the whole year, while Figure 20(b) displays the ratio of the summer (mean of June, July and August) to winter (mean of January, February and March) values of γ_v. The regional differences are most clearly visible in Figure 20(b), with high values over lowland England and low values in eastern Scotland and north-east England.

6.10. *Procedure for obtaining objective estimates of return periods of areal rainfall events*

Return periods of areal rainfall events lasting n calendar months and starting in a given month can be estimated objectively from the following input data:

(a) Point values of C_v and C_s of monthly rainfall, meaned over all months, and averaged over uniformly distributed points within the area.
(b) Mean monthly rainfall over the area.
(c) Mean distance of uniformly distributed points within the area from the centre of the area (d).
(d) Areal reduction coefficients of the C_v of monthly rainfall, averaged over all months ($\gamma_{v,y}$), and over June, July and August ($\gamma_{v,s}$).

The coefficient of variation of rainfall over an area A for a duration of n months is then given by:

$$C_{v(n,\bar{R},A)} = C_{v(1,r,0)} \left(\frac{1}{n^{0.48}}\right) \left(\frac{nr}{\bar{R}}\right)^{\beta_t} \exp(-\gamma_{v,n} d^{0.8}) \qquad \ldots (26)$$

where β_t is given by equation (17) and $\gamma_{v,n}$ by equations (20) and (25). A similar equation is used for C_s, with

$$\gamma_{s,n} = 2.5\, \gamma_{v,n}.$$

The calculated values of $C_{v(n,\bar{R},A)}$ and $C_{s(n,\bar{R},A)}$ are then put in to the modified log-normal distribution described in the Appendix to estimate the appropriate areal rainfall distribution.

6.11. *Comparison with 'observed' results*

Comparisons between the modelled and 'observed' distributions of rainfall over England and Wales as a whole and over the south-eastern quarter thereof are presented in Table IX and Figure 21. The 'observed' rainfalls were obtained from an areal rainfall record constructed by the method described in subsection 6.1 and analysed by the method described in section 3. Forty-eight stations were used to cover the whole of England and Wales, and 24 stations for the south-eastern quarter, for the period 1911–70. The figures presented in Table IX are for durations of one month and 12 months and are meaned over all starting months. The agreement between the two sets of figures is in general good. For a duration of 12 months, however, the 'observed' rainfalls are negatively skewed, a consequence of the inadequate statistical sampling to which attention was first drawn in section 3.

Seasonal variations are examined in Figure 21 by comparing the modelled and the 'observed' distributions of rainfall for individual calendar months. The parameters examined are the coefficient of variation and the percentage of average rainfall corresponding to rainfall excesses and deficiencies expected to be reached or exceeded on average once every 50 years. The 'observed' values exhibit large random variations (see Figures 2 and 4), but seasonal differences are in general small and plausibly represented by the modelled results.

TABLE IX. *Percentage of average rainfall corresponding to given return periods*

Return period	England and Wales				England and Wales (SE quarter)			
	1 month		12 months		1 month		12 months	
years	obs.	mod.	obs.	mod.	obs.	mod.	obs.	mod.
+50	202	198	126	128	219	213	131	132
+20	180	176	121	122	193	187	125	125
+10	159	157	117	117	167	165	120	119
+5	135	135	112	111	139	139	114	112
−2	97	97	100	100	95	96	99	100
−5	63	63	88	89	58	58	88	87
−10	46	47	83	83	39	41	81	81
−20	33	34	77	79	27	27	73	76
−50	22	21	71	74	16	15	65	71

Comparison between observed (obs.) and modelled (mod.) distributions of areal rainfall for period 1911–70, meaned over all starting months. Positive return periods refer to rainfall excesses and negative return periods refer to rainfall deficiencies.

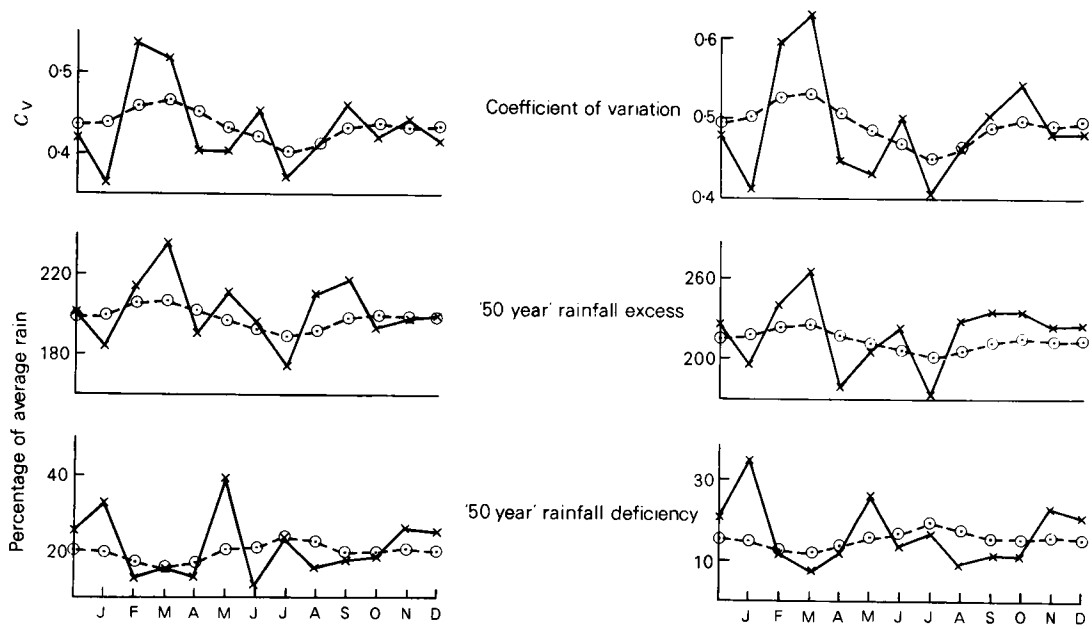

FIGURE 21. Seasonal variations in the distribution of monthly rainfall

Comparison between observed and calculated values of the coefficient of variation and the percentage of average rainfall corresponding to rainfall excesses and deficiencies with a return period of 50 years. Figures are based on the period 1911–70.

x———x Observed values; o – – – o Calculated values.

(a) England and Wales (b) South-east quarter of England and Wales

For the whole of England and Wales and its south-eastern quarter, Table X compares the modelled areal rainfall distribution with the modelled point rainfall distribution. The differences between the figures are greater for a duration of one month than for a duration of 12 months and are greater for rare events than for common events. However, on the whole the differences between the figures are not great.

TABLE X. *Percentage of average rainfall corresponding to given return periods*

Return period	England and Wales				England and Wales (SE quarter)			
	1 month		12 months		1 month		12 months	
years	point	area	point	area	point	area	point	area
+200	269	228	144	135	277	250	146	140
+100	248	213	140	132	255	232	141	136
+50	226	198	135	128	232	213	136	132
+20	195	176	127	122	199	187	128	125
+10	170	157	121	117	173	165	122	119
+5	142	135	113	111	143	139	114	112
−2	95	97	99	100	94	96	99	100
−5	55	63	86	89	53	58	86	87
−10	37	47	80	83	35	41	79	81
−20	23	34	74	79	22	27	73	76
−50	13	21	69	74	11	15	67	71
−100	8	14	65	71	7	10	64	67
−200	5	10	61	68	4	6	60	64

Comparison between modelled distributions of point and areal rainfall for the period 1911–70, meaned over all starting months. Positive return periods refer to rainfall excesses and negative return periods refer to rainfall deficiencies.

6.12. *Confidence limits*

In subsection 5.6, in investigating the confidence limits of the modelled point rainfall distribution, the order of importance of the various input parameters was found to be R_A, α, C_v and C_s. For areal rainfall, the effects of γ_v have also to be considered.

The closeness of the rainfall distribution over an area to that over a point ensures that the results are relatively insensitive to the areal reduction coefficient. Table X shows that over the south-eastern quarter of England and Wales, for a duration of 12 months, the '200-year' rainfall deficiency increases from 60 per cent of average at a point to 64 per cent of average over the whole area. In section 6 it was pointed out that the standard error of γ_v could not be properly estimated. Even if γ_v were inaccurate to a factor of almost 2, however, the practical effect would only be to cause the estimated areal '200-year' rainfall deficiency to range from 63 per cent to 65 per cent of average. Thus it can be seen that the accuracy of the modelled areal rainfall distribution depends far more on the corresponding point rainfall distribution than on the areal reduction coefficient. The modelled rainfall distribution over an area will therefore be less accurate than that over a point but the decrease in accuracy is not easy to quantify and will not be very great.

CONCLUSIONS

After summing up the results of this investigation, the following conclusions were reached:

(a) This paper presents a technique for calculating the return periods of rainfall excesses or deficiencies, measured over n months starting in month m, for any point in Great Britain.

(b) For durations of one month and more the observed rainfall distribution over Great Britain can be adequately represented by the log-normal distribution. Some modification is necessary where the 'dry' tail approaches zero rainfall.

(c) In confirmation of previous findings, consecutive monthly rainfalls were found to be independent of one another.

(d) For durations of one month and more, C_v and C_s showed only small seasonal variations.

(e) C_v increased with continentality, being lowest in maritime areas and highest to the orographic lee of the prevailing wind. When this geographical variation was taken into account there was little dependence of C_v upon R.

(f) As a result of (c) and (d), C_v over n months was approximately $(1/\sqrt{n}) \times C_v$ over one month.

(g) As a result of (f) applying to both points and areas, the areal reduction coefficient of C_v was found to be independent of duration (for durations of one month or more).

(h) Large geographical and seasonal variations in the areal reduction coefficient are caused by the combined influences of convection and relief. Summer reduction coefficients are determined mainly by the intensity of convection, while winter values are determined mainly by topography. In winter, the highest reduction coefficients are found on the leeward slopes of hills (in relation to the prevailing wind).

(j) Areal reduction coefficients are in general small. As the size of an area is expanded from a single point to cover the whole of England and Wales, C_v is decreased by only about 20 per cent.

(k) For typical durations of interest, point rainfall amount (derived from the model) corresponding to given return periods can be regarded as being accurate to ± 4 per cent, whereas return periods corresponding to given rainfall amount may only be accurate to a factor of 2. Uncertainties in estimates of areal rainfall events are likely to be similar to those relating to point rainfall. For sensitivity to input parameters see 5.6 and 6.12.

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APPENDIX

Consider a set of N observations Z_i of a variable Z with a mean value $\bar{Z}$. The variance (σ^2) of Z is defined as

$$\sigma^2 = \sum_{i=1}^{N} (Z_i - \bar{Z})^2 / (N - 1)$$

The coefficient of variation (V) of Z is defined as $V = \sigma/\bar{Z}$ where σ is known as the standard deviation of Z.

The coefficient of skewness (S) of Z is defined as

$$S = \sum_{i=1}^{N} (Z_i - \bar{Z})^3 / N\sigma^3$$

Let x represent the fractional deviation of rainfall from average

i.e. $$x = \frac{R - \bar{R}}{\bar{R}}$$

Mean value of x	= 0	$(\bar{x})$
Standard deviation of x	= coefficient of variation of R	(C_v)
Coefficient of skewness of x	= coefficient of skewness of R	(C_s)

Let f(x) denote the probability density function of x

Let l represent the lower bound for f(x), i.e. the lower value of x for which f(x) = 0.

Then, in the three-parameter log-normal distribution, $\ln(x - l)$ is normally distributed with mean μ_n and standard deviation σ_n,

i.e. $$f(x) = \frac{1}{(x - l)\sigma_n\sqrt{(2\pi)}} . \exp\left\{-[\ln(x - l) - \mu_n]^2 / 2\sigma_n^2\right\} \quad \text{(A.1)}$$

with $$\bar{x} = l + \exp(0.5\sigma_n^2 + \mu_n) \quad \text{(A.2)}$$

$$C_v^2 = \exp[2(\sigma_n^2 + \mu_n)] - \exp(\sigma_n^2 + 2\mu_n) \quad \text{(A.3)}$$

and $$C_s = \left\{\exp[3\sigma_n^2] - 3\exp[\sigma_n^2] + 2\right\} / \left\{\exp[\sigma_n^2] - 1\right\}^{1.5} \quad \text{(A.4)}$$

Equations (A.1) to (A.4) are given by Wallis *et alii* (1974). The parameters μ_n, σ_n, and l in equation (A.1) can be obtained from C_v and C_s from

$$\omega = [0.5C_s + \sqrt{(0.25C_s^2 + 1)}]^{\frac{1}{3}} + [0.5C_s - \sqrt{(0.25C_s^2 + 1)}]^{\frac{1}{3}} \quad \ldots \text{(A.5)}$$

$$\sigma_n^2 = \ln(l + \omega^2) \quad \ldots \text{(A.6)}$$

$$\mu_n = \ln C_v - 0.5 \ln(\exp 2\sigma_n^2 - \exp \sigma_n^2) \quad \ldots \text{(A.7)}$$

$$l = -\exp(0.5\sigma_n^2 + \mu_n) \quad \ldots \text{(A.8)}$$

Equations (A.5) and (A.6) are quoted by Brooks and Carruthers (1953), while equations (A.7) and (A.8) are rearrangements of equations (A.3) and (A.2) respectively.

The cumulative frequency distribution $F(x)$ was calculated from $f(x)$ by a three-point Gauss quadrature method, as described by McCracken and Dorn (1964). At the dry end of the spectrum the values of x corresponding to given value of $F(x)$ were amended to $x + \Delta x$ in order to obtain the best fit with the data for Kew for 1697 to 1973. Values of Δx, obtained by the method of least squares, are given by:

$\Delta x = 0$	for $-0.7 \leqslant x$
$\Delta x = 0.889x^2 + 1.244x + 0.437$	for $-1.2 < x < -0.7$
$\Delta x = -0.9x - 0.856$	$x \leqslant -1.2$

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